Power System Calculations—Part II

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Introduction

- Topics
  - Per Unit Basis
  - Symmetrical Components
  - Component Modelling
  - Faults & Sequence Networks

Per Unit Basis

Per Unit Conversion

- Per Unit Definition

A quantity a (voltage, current, power, impedance, admittance, etc.) in per unit is defined as the ratio of that quantity to a selected base quantity of the same nature (i.e. voltage, current, power, impedance, admittance, etc.)

$$a^{[\text{per unit}]} = \frac{a^{[\text{actual units}]}}{a_b^{[\text{actual units}]}} \Rightarrow a^{[\text{pu}]} = \frac{a^{[\text{units}]}}{a_b^{[\text{units}]}}$$

Per Unit

- Example:

If $V = 120$ [V] and we select $V_b = 80$ [V], then

$$V^{[\mathrm{pu}]} = \frac{120\ [\mathrm{V}]}{80\ [\mathrm{V}]} = 1.5\ [\mathrm{pu}]$$

Per Unit

- Percent Definition

  If the per unit value is multiplied by 100, the quantity is expressed in percent with respect to the base quantity.

  For the previous example, we say the voltage is 150% (1.5 x 100) of the base voltage of 80 [V].

Per Unit

- Per unit conversion requires us to select a base quantity
- How do we make the selection?
  - Answer: Select two quantities as the base from the following: voltage, current, power, impedance, admittance
- Which do we choose?
  - Answer: Generally choose voltage and power.

Per Unit

- Why Voltage and Power?
  - *Voltage*. For each voltage level in our system, we know the rated voltage of equipment, and even if loading changes, the voltage does not deviate too much from the rated value.
  - *Power*. The range of power flowing in a section of the system is quadratically related with the voltage. As such, the range of expected power flow is known for an area. Note, for transmission level analysis, it is customary to select a base power of 100 MVA. *Note:* The base power is usually selected to be the same for the entire network.

Conventions

- Unless specifically stated, the following quantities are always complex numbers:
  V, I, Z, Y, S, t
- All other quantities are real numbers, such as
  R, X, G, B, L, C, P, Q

Terminology

V	Voltage
I	Current
Z	Impedance
Y	Admittance
S	Complex Power
t	Turn Ratio
R	Resistance
X	Reactance
G	Conductance
B	Susceptance

L	Inductance
C	Capacitance
P	Active Power
Q	Reactive Power
$\|S\|$	Apparent Power

- Pet Peeve #1: There is no “real power" anywhere here

Per Unit

- Selecting Quantities

$$
\begin{aligned}
V_b &= V_{\phi-\phi} \\
S_b &= S_{3\phi}
\end{aligned}
$$

  - *Voltage*
    - Usually selected as the nominal phase-to-phase voltage at each voltage level
  - *Power*
    - Usually selected in the range of 3-phase power flowing in the network (i.e. whatever network is being analyzed)
    - For transmission level analysis, it is customary to select a base power of 100 MVA.
    - The base power is usually selected to be the same for the entire network.

Per Unit Formulas

$$I_{\phi-n\bullet b}^{[\mathrm{A}]} = \frac{S_{3\phi\bullet b}^{[\mathrm{VA}]}}{\sqrt{3}V_{\phi-\phi\bullet b}^{[\mathrm{V}]}}, \qquad V_{\phi-n\bullet b}^{[\mathrm{V}]} = \frac{V_{\phi-\phi\bullet b}^{[\mathrm{V}]}}{\sqrt{3}}$$

$$Z_{b}^{[\Omega]} = \frac{\left(V_{\phi-\phi\bullet b}^{[\mathrm{V}]}\right)^{2}}{S_{3\phi\bullet b}^{[\mathrm{VA}]}}, \qquad Y_{b}^{[\mathrm{Siemens}]} = \frac{1}{Z_{b}^{[\Omega]}}$$

Per Unit for Equipment

- Per unit quantities for equipment is defined at the equipment level
  - Ex: 25 MVA, 110-33 kV transformer
    Z = 8.3%
  - Ex: 4 kV, 1500 hp, 0.88 PF, 93.5% eff
    $X_d'' = 0.155$ pu

$$S_{\text{motor}} = S_b = \frac{1500\ [\text{hp}] \times 0.746\ [\text{hp/kW}]}{0.88\ \text{PF} \times 0.935\ \text{eff}}$$
$$= 1360\ [\text{kVA}]$$

Per Unit Conversion

- Equipment ratings vary
- Selection of base quantities vary
- Analysis requires use of a common base

$$A^{[\text{pu.old base}]} = \frac{A^{[\text{units}]}}{A_{\text{b.old}}^{[\text{units}]}}; \quad A^{[\text{pu.new base}]} = \frac{A^{[\text{units}]}}{A_{\text{b.new}}^{[\text{units}]}}$$

$$A^{[\text{pu.old base}]} A_{\text{b.old}}^{[\text{units}]} = A^{[\text{pu.new base}]} A_{\text{b.new}}^{[\text{units}]}$$

$$A^{[\text{pu.new base}]} = A^{[\text{pu.old base}]} \frac{A_{\text{b.old}}^{[\text{units}]}}{A_{\text{b.new}}^{[\text{units}]}}$$

Per Unit Conversion

- Particularly for impedances
  Remember:

$$Z_b^{[\Omega]} = \frac{\left(V_b^{[\mathrm{V}]}\right)^2}{S_b^{[\mathrm{VA}]}}$$

$$Z^{[\text{pu.new base}]} = Z^{[\text{pu.old base}]}\left(\frac{Z_{\text{b.old}}^{[\Omega]}}{Z_{\text{b.new}}^{[\Omega]}}\right)$$

$$= Z^{[\text{pu.old base}]}\left(\frac{V_{\text{b.old}}^{[\mathrm{V}]}}{V_{\text{b.new}}^{[\mathrm{V}]}}\right)^2\left(\frac{S_{\text{b.new}}^{[\mathrm{VA}]}}{S_{\text{b.old}}^{[\mathrm{VA}]}}\right)$$

Advantages Of Per Unit

- *Equipment Parameters*. For a given type of equipment, and disregarding the size and voltage, the parameters in per unit are within a narrow, known range
- *Eliminate Turn Ratio*. For two adjacent networks of different voltage levels, if the selected base power is the same throughout and the selected base voltages match the turn ratio of the transformer between the networks, then all quantities in per unit have the same value regardless of which voltage level they are defined. In essence, the transformer is eliminated.
- *Eliminate Coefficients*. For almost all equations with quantities defined in per unit, the numerical coefficients are eliminated.
- *Voltage*. In per unit, the line-to-neutral voltage equals the phase-to-phase voltage, and during normal operation both quantities are close to unity.

Symmetrical Components

History

- Charles LeGeyt Fortescue
  - 1876 – Born in York Factory, Manitoba
  - 1898 – First engineer to graduate from Queens University at Kingston in Ontario
  - Joined Westinghouse after graduation and spent his entire career there
  - 1913 – Co-authored paper on measurement of high voltage using sphere gap, a method still used to this day (97 years later)
  - Obtained 185 patents in his career in design transformers, insulators, and DC and AC power circuits

History

- Charles LeGeyt Fortescue (cont'd)
  - 1921 – Elected AIEE Fellow
  - 1930 – Paper in *Electric Journal* that outlined "direct stroke theory", which is said to have completely revolutionized the approach to the lightning problem. Led to adoption of overhead static lines.
  - 1936 – Died in December at age 60

History

- Charles LeGeyt Fortescue (cont'd)
  - 1918 - *Transactions of the American Institute of Electrical Engineers* (AIEE) included the classic paper *Method of Symmetrical Co-Ordinates Applied to the Solution of Polyphase Networks*
    - 88 pages long
    - 24 additional pages of discussion
    - 303 numbered equations
    - "bewildering exhibit of subscripts to be found in it is something that will, well, make one pause"
    - Vladimir Karapetoff suggested that the term "symmetrical components" was a "more correct and descriptive" expression

Symmetrical "Co-Ordinates"

- What are Symmetrical Components?
  - Any set of *N* unbalanced phasors — that is, any such "polyphase" signal — can be expressed as the sum of *N* symmetrical sets of balanced phasors.
  - Only a single frequency component is represented by the phasors. This is overcome by using techniques such as Fourier or LaPlace transforms.
  - Absolutely general and rigorous and can be applied to both steady state and transient problems.
  - It is thoroughly established as preeminently the only effective method of analyzing general polyphase network problems

Three-Phase Sequence Networks

- Three-Phase Systems
  - Three sets of symmetrical components, where each set is referred to as a *sequence*.
  - First set of phasors, called the *positive sequence*, has the same phase sequence as the system under study (say ABC)
  - The second set, the *negative sequence*, has the reverse phase sequence (BAC)
  - The third set, the *zero sequence*, phasors A, B and C are in phase with each other.
  - Method converts *any* set of three phasors into three sets of symmetrical phasors, which makes asymmetric analysis more tractable.

Symmetrical Components

- Set of three phasors, say X_a, X_b and X_c can be represented as a sum of the three sequence sets

$$\begin{aligned} X_a &= X_{a0} + X_{a1} + X_{a2} \\ X_b &= X_{b0} + X_{b1} + X_{b2} \\ X_c &= X_{c0} + X_{c1} + X_{c2} \end{aligned}$$

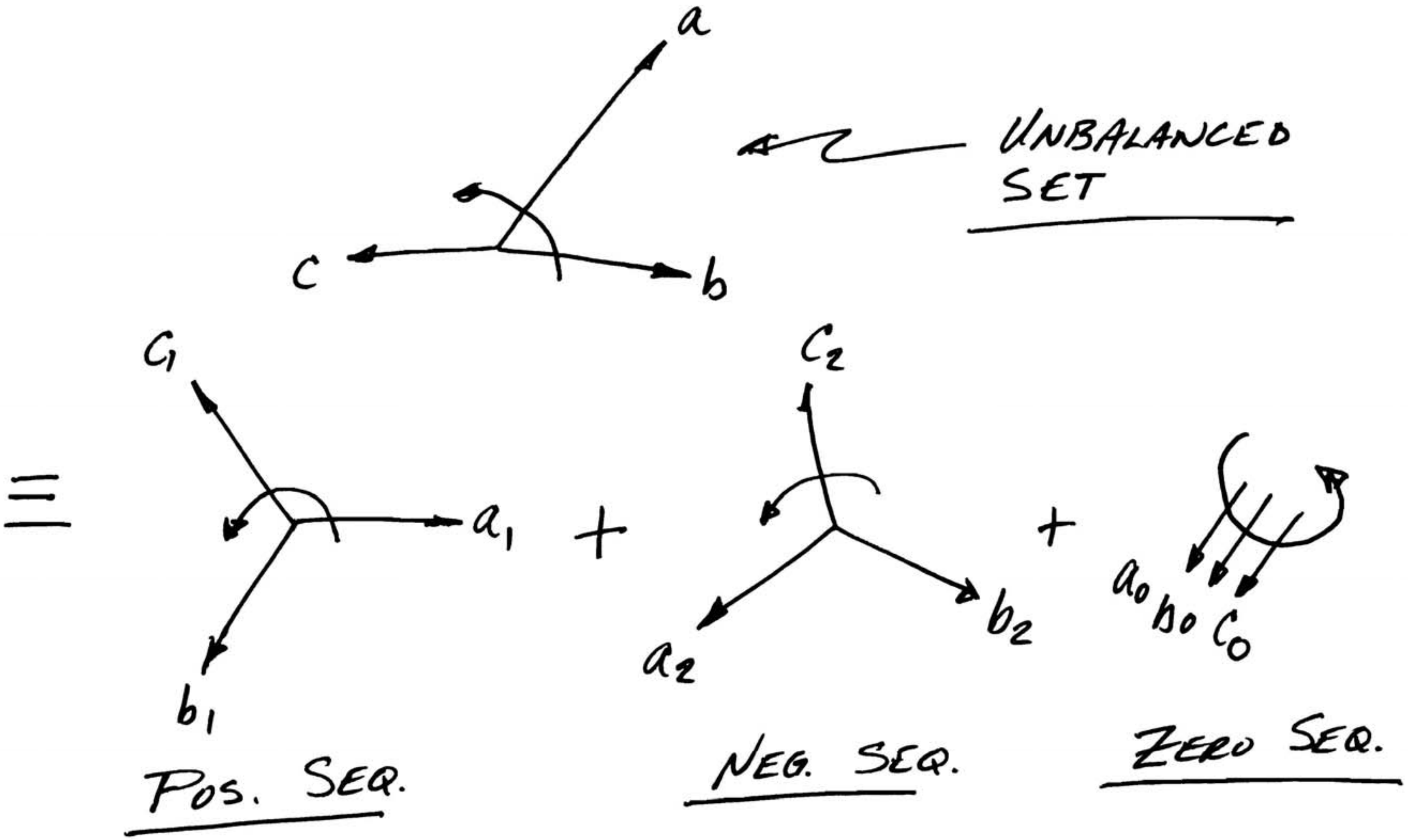
where

X_{a0}, X_{b0}, X_{c0} is the zero sequence set

X_{a1}, X_{b1}, X_{c1} is the positive sequence set

X_{a2}, X_{b2}, X_{c2} is the negative sequence set

Symmetrical Components



Symmetrical Components

Only three of the sequence values are unique,

$$X_a, X_{a1}, X_{a2}$$

The others can be determined. First, we define a complex operator a.

$$a \triangleq e^{j120^\circ} = 1\angle 120^\circ$$
$$a^2 = e^{j240^\circ} = 1\angle 240^\circ$$
$$a^3 = e^{j360^\circ} = 1\angle 360^\circ = 1$$
$$1 + a + a^2 = 0$$

Symmetrical Components

Then, we relate phases within a sequence to the reference, which is a-phase in this example

$$
\begin{array}{llll}
X_{a0} : & X_{b0} = X_{a0}, & X_{c0} = X_{a0} \\
X_{a1} : & X_{b1} = a^2 X_{a1} & X_{c1} = aX_{a1} \\
X_{a2} : & X_{b2} = aX_{a2} & X_{c2} = a^2 X_{a2}
\end{array}
$$

Symmetrical Components

Rearrange the equations

$$
\begin{aligned}
X_a &= X_{a0} + X_{a1} + X_{a2} \\
X_b &= X_{a0} + a^2 X_{a1} + a X_{a2} \\
X_c &= X_{a0} + a X_{a1} + a^2 X_{a2}
\end{aligned}
$$

which in matrix form looks like this:

$$
\begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} X_{a0} \\ X_{a1} \\ X_{a2} \end{bmatrix}
$$

Symmetrical Components

Using simple notation, we can convert from symmetrical component reference to the phase reference as follows:

$$\mathbf{X}_{\text{phase}} = \mathbf{T}\mathbf{X}_{\text{sym}}$$

where

$$\mathbf{T} \triangleq \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

Of course, the following equation can convert from the phase reference to the symmetrical component reference:

$$\mathbf{X}_{\text{sym}} = \mathbf{T}^{-1}\mathbf{X}_{\text{phase}}$$

Symmetrical Components

The inverse of the transformation matrix is

$$\mathbf{T}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

which in detail, results in the following equations:

$$\begin{bmatrix} X_{a0} \\ X_{a1} \\ X_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix}$$

Sym. Components – Example 1

Let

$$\mathbf{I} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 10\angle -120° \\ 10\angle 120° \end{bmatrix}$$

Then

$$\begin{aligned} \mathbf{I}_s &= \mathbf{T}^{-1}\mathbf{I} \\ &= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 10\angle 0° \\ 10\angle -120° \\ 10\angle 120° \end{bmatrix} = \begin{bmatrix} 0 \\ 10\angle 0° \\ 0 \end{bmatrix} \end{aligned}$$

Sym. Components – Example 2

If

$$\mathbf{I} = \begin{bmatrix} 10\angle 0^\circ \\ 10\angle +120^\circ \\ 10\angle -120^\circ \end{bmatrix}$$

Then

$$\mathbf{I}_s = \begin{bmatrix} 0 \\ 0 \\ 10\angle 0^\circ \end{bmatrix}$$

Sym. Components – Example 3

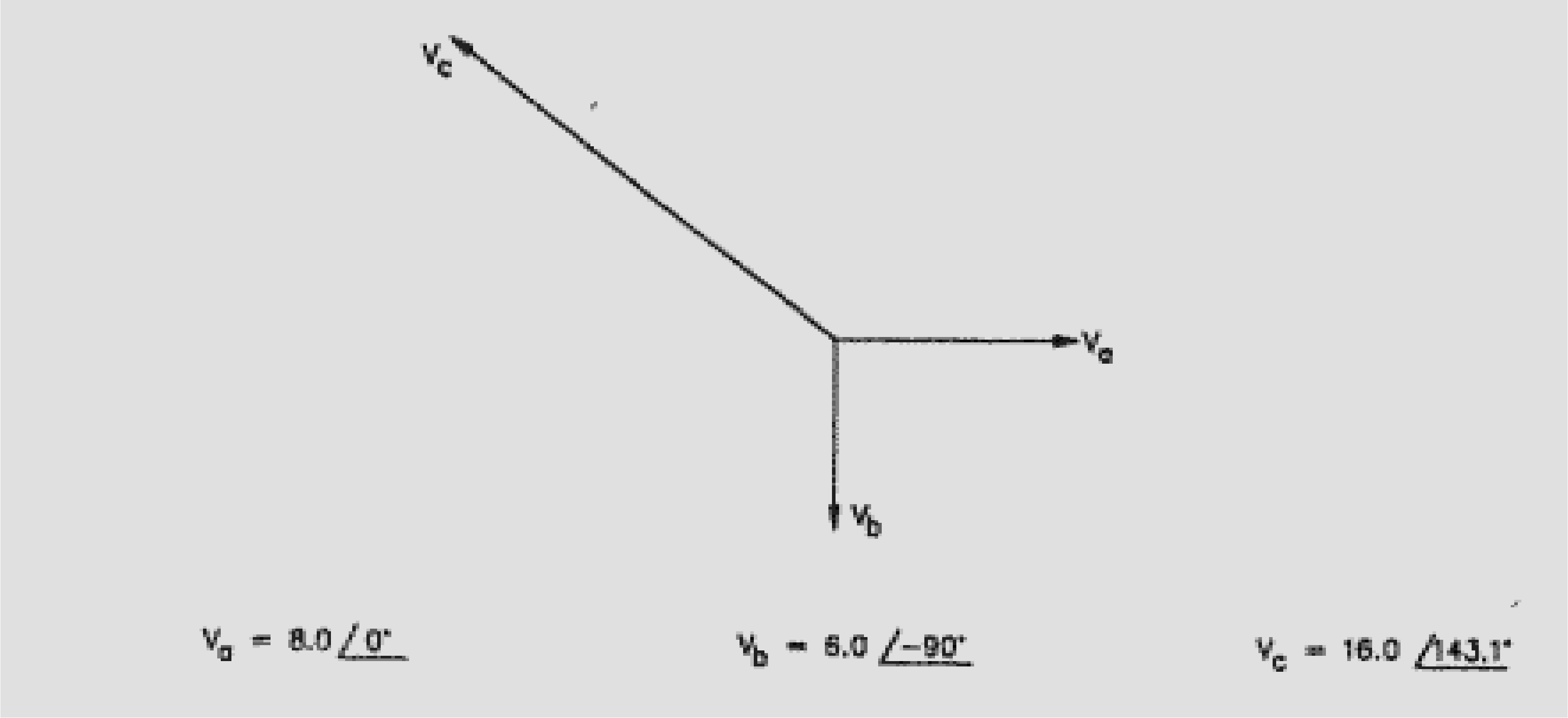
If

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 8.0\angle 0^\circ \\ 6.0\angle -90^\circ \\ 16.0\angle 143.1^\circ \end{bmatrix}$$

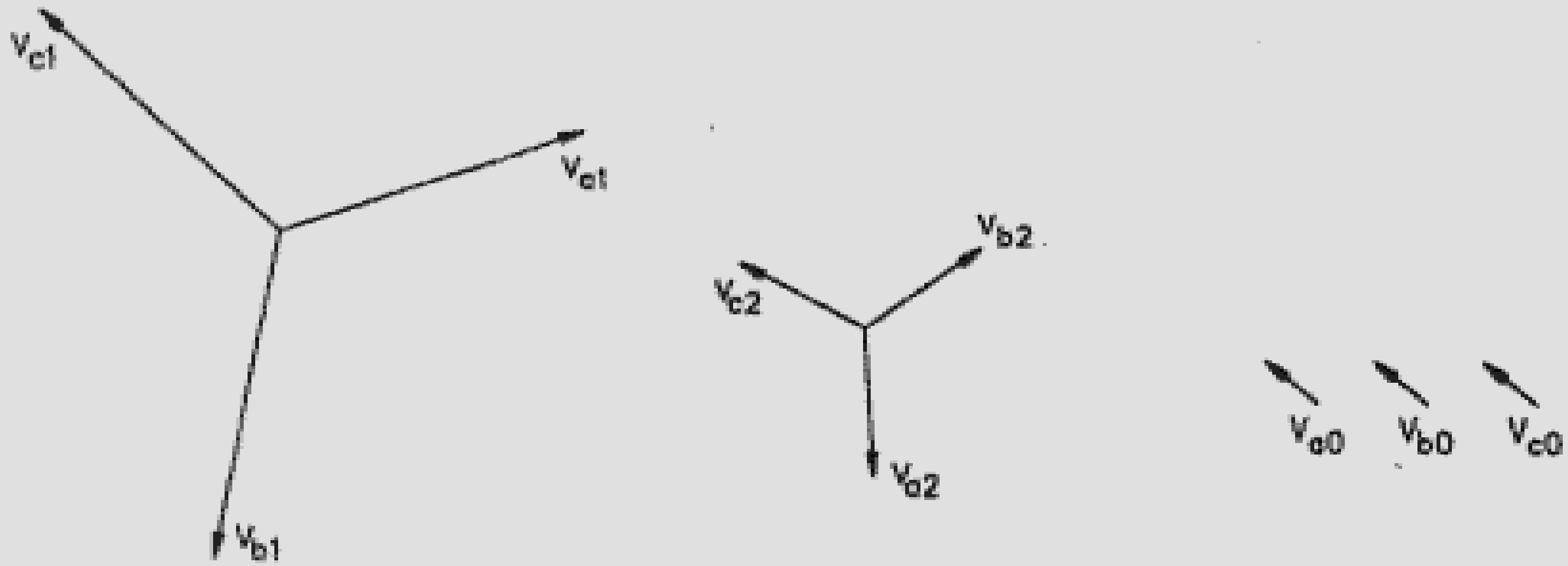
Then

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2.0\angle 143.1^\circ \\ 9.8\angle 18.4^\circ \\ 4.3\angle -86.2^\circ \end{bmatrix}$$

Symmetrical Components

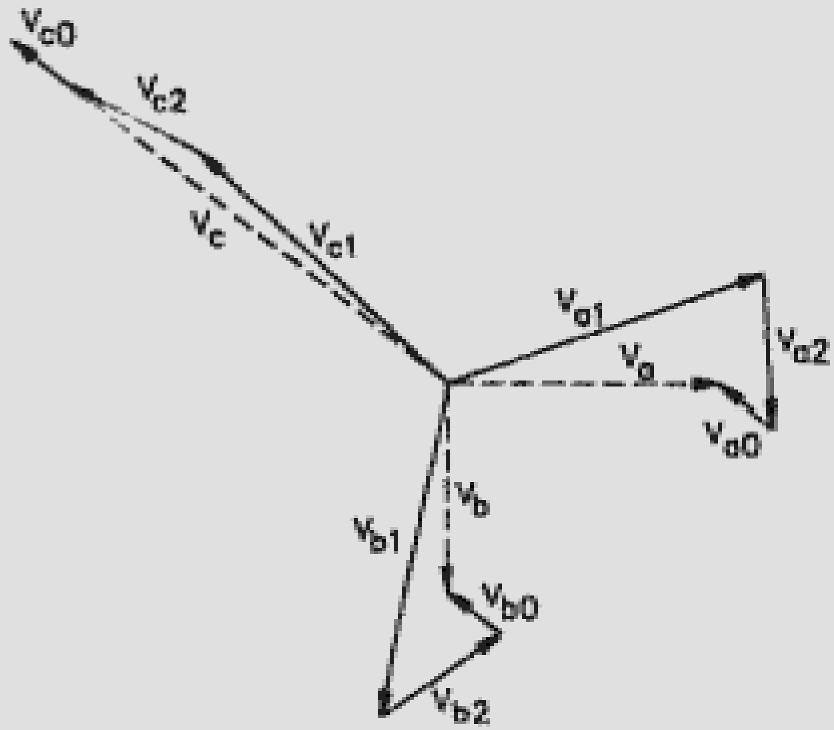


Symmetrical Components



$V_{a1} = 9.8 \angle 18.4^\circ$	$V_{a2} = 4.3 \angle -86.2^\circ$	$V_{a0} = 2.0 \angle 143.1^\circ$
$V_{b1} = 9.8 \angle -101.6^\circ$	$V_{b2} = 4.3 \angle 33.8^\circ$	$V_{b0} = 2.0 \angle 143.1^\circ$
$V_{c1} = 9.8 \angle 138.4^\circ$	$V_{c2} = 4.3 \angle -206.2^\circ$	$V_{c0} = 2.0 \angle 143.1^\circ$

Symmetrical Components



One Way to Visualize

- Positive Sequence
  - Can be thought of as a “source” that rotates a machine in a particular direction
- Negative Sequence
  - This “source” rotates a machine in a direction opposite of that of the positive sequence
- Zero Sequence
  - Causes no rotational force, but instead forms an oscillating (not rotating) field in the machine

Comments on Sym. Components

- The transformations apply only for linear systems, that is, systems with constant parameters (impedance, admittance) independent of voltages and currents
- The quantities used for X can be phase-to-neutral or phase-to-phase voltages, or line or line-to-line currents.
- For some connections, the zero sequence component is always zero
  - Line currents for an ungrounded wye connection
  - Line currents for a delta connection
  - Line-to-line voltages for a delta connection

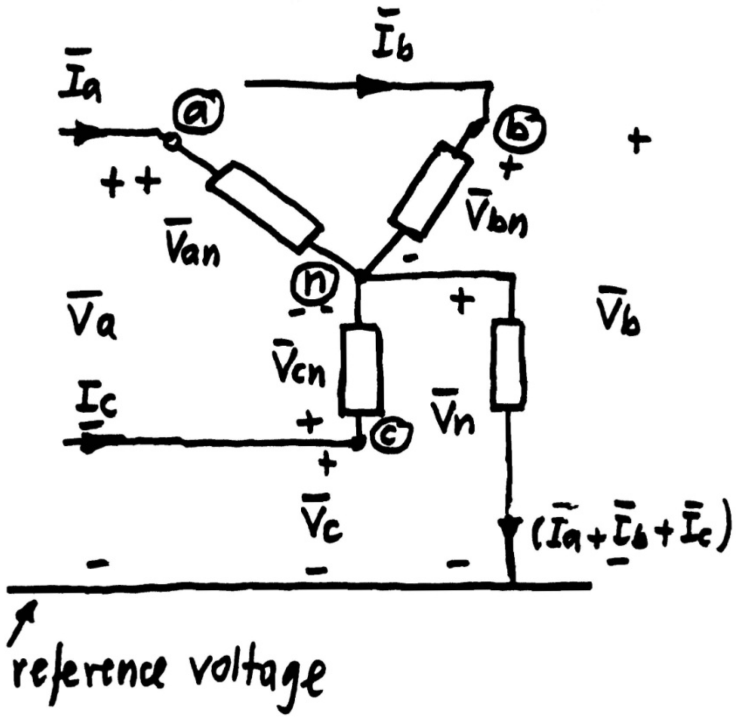
More Comments

- Symmetrical components is a important tool for analyzing unbalanced states
  - Three-phase unsymmetrical network and state is converted to three symmetrical networks and states
  - Symmetrical networks can be solved using single phase techniques
  - With symmetrical components we solve three interconnected symmetrical networks using single phase analysis, which is easy.
  - Once solved, we use transformation equations to obtain phase quantities

More Comments

- Equipment Parameters
  - Symmetrical components has advantage that parameters in system components are easier to define
  - Because each sequence is a symmetrical three-phase case, the parameters can be defined using typical three-phase tests.

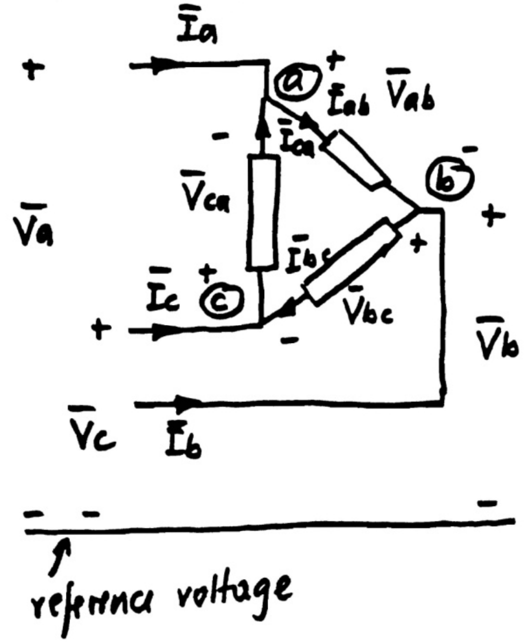
Complex Power with Symmetrical Components



For wye connections

$$
\begin{aligned}
S &= V_{an}I_a^* + V_{bn}I_b^* + V_{cn}I_c^* + V_n\left(I_a + I_b + I_c\right)^* \\
&= \left(V_{an} + V_n\right)I_a^* + \left(V_{bn} + V_n\right)I_b^* + \left(V_{cn} + V_n\right)I_c^* \\
&= V_a I_a^* + V_b I_b^* + V_c I_c^*
\end{aligned}
$$

Complex Power with Symmetrical Components



For delta connections

$$
\begin{aligned}
S &= V_{ab} I_{ab}^* + V_{bc} I_{bc}^* + V_{ca} I_{ca}^* \\
&= \left(V_a - V_b\right) I_{ab}^* + \left(V_b - V_c\right) I_{bc}^* + \left(V_c - V_a\right) I_{ca}^* \\
&= V_a \left(I_{ab} - I_{ca}\right)^* + V_b \left(I_{bc} - I_{ab}\right)^* + V_c \left(I_{ca} - I_{bc}\right)^* \\
&= V_a I_a^* + V_b I_b^* + V_c I_c^*
\end{aligned}
$$

Complex Power with Symmetrical Components

In matrix form

$$S = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \mathbf{V}_{\text{phase}}^{\mathrm{T}} \mathbf{I}_{\text{phase}}^*$$

We have the following transforms

$$\mathbf{V}_{\text{phase}} = \mathbf{T}\mathbf{V}_{\text{sym}}$$

$$\mathbf{I}_{\text{phase}} = \mathbf{T}\mathbf{I}_{\text{sym}}$$

Then the complex power becomes

$$S = \mathbf{V}_{\text{sym}}^{\mathrm{T}} \mathbf{T}^{\mathrm{T}} \mathbf{T}^* \mathbf{I}_{\text{sym}}^*$$

Complex Power with Symmetrical Components

The product

$$\mathbf{T}^{\mathrm{T}}\mathbf{T}^{*} = 3\mathbf{I}_{\text{identity}} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

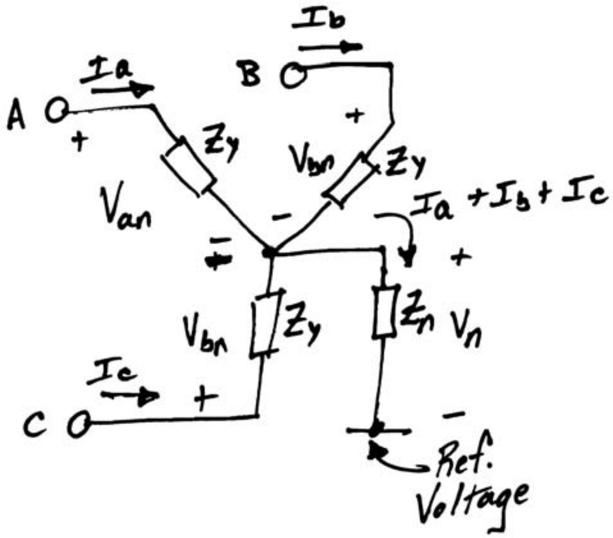
Then the complex power becomes

$$\begin{aligned} S &= 3\mathbf{V}_{\text{sym}}^{\mathrm{T}}\mathbf{I}_{\text{sym}}^{*} \\ &= 3\left(V_{a0}I_{a0}^{*} + V_{a1}I_{a1}^{*} + V_{a2}I_{a2}^{*}\right) \end{aligned}$$

If the voltages and current are defined in per unit, the "3" disappears

Use of Symmetrical Components

- Consider the following wye connected load



$$I_n = I_a + I_b + I_c$$
$$V_a = Z_Y I_a + Z_n I_n$$
$$V_a = (Z_Y + Z_n) I_a + Z_n I_b + Z_n I_c$$
$$V_b = Z_n I_a + (Z_Y + Z_n) I_b + Z_n I_c$$
$$V_c = Z_n I_a + Z_n I_b + (Z_Y + Z_n) I_c$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Use of Symmetrical Components

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\mathbf{V} = \mathbf{ZI}$$

$$\mathbf{V} = \mathbf{TV}_{\text{sym}}$$

$$\mathbf{I} = \mathbf{TI}_{\text{sym}}$$

$$\mathbf{TV}_{\text{sym}} = \mathbf{ZTI}_{\text{sym}}$$

$$\mathbf{V}_{\text{sym}} = \mathbf{T}^{-1}\mathbf{ZTI}_{\text{sym}}$$

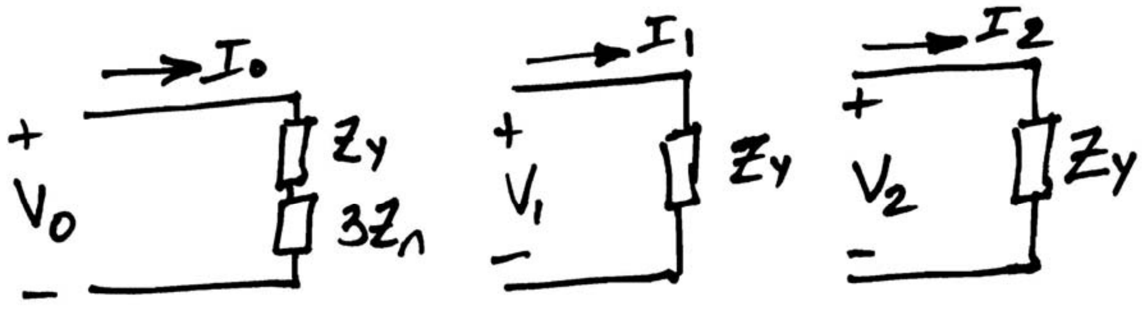
$$\mathbf{T}^{-1}\mathbf{ZT} = \begin{bmatrix} Z_y + 3Z_n & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix}$$

Use of Symmetrical Components

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_Y + 3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

Systems are decoupled

$$V_0 = (Z_Y + 3Z_n)\, I_0$$
$$V_1 = Z_Y\, I_1$$
$$V_2 = Z_Y\, I_2$$



Break

Component Modelling

Generators

- Fault Behavior
  - Sudden change in voltage and current, such as those in faults, produces transients
  - Armature current divided into two components
    - Symmetrical AC component – whose associated component in the field is a DC current
    - DC component – whose associated component in the field is an AC current

Generators

- Symmetrical Component Modelling
  - Principle concern is with symmetrical component and its associated constants
  - DC component often eliminated from studies
    - Usually not necessary to apply or set protective relays
    - If necessary (e.g. circuit breaker applications), various factors are available from standards, manufacturers, or other sources
  - For synchronous machines, symmetrical AC component can be resolved into three distinct components
    - Subtransient component – the double prime (“) values
    - Transient component – the single prime (‘) values
    - The steady-state component

Generators

- Subtransient Component
  - Occurs during onset of fault
  - Subtransient reactance (X_d") approaches armature leakage reactance but is higher as a result of damper windings, and so on.
  - Subtransient time constant (T_d") is very low (because damper windings have relatively high resistance), typically around 0.01–0.05 seconds

Generators

- Transient Component
  - Armature current demagnetizes the field and decrease flux linkages with the field winding
  - Transient reactance (X_d') includes effect of both armature and field leakages and is higher than armature leakage reactance, and thus higher than the subtransient reactance
  - Transient time constant (T_d') varies typically from 0.35 to 3.3 seconds
- Steady-State Component
  - Transient eventually decays
  - For faults, eventually becomes unsaturated direct axis reactance (X_d)

Generators

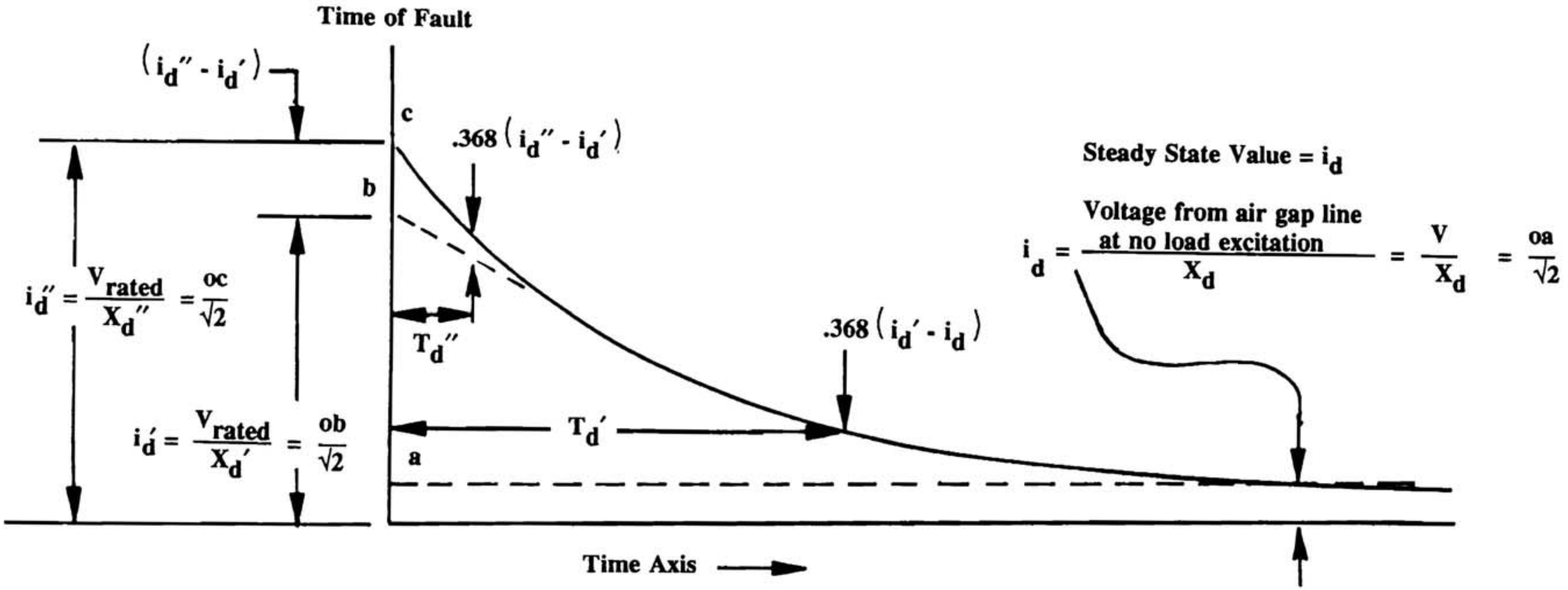


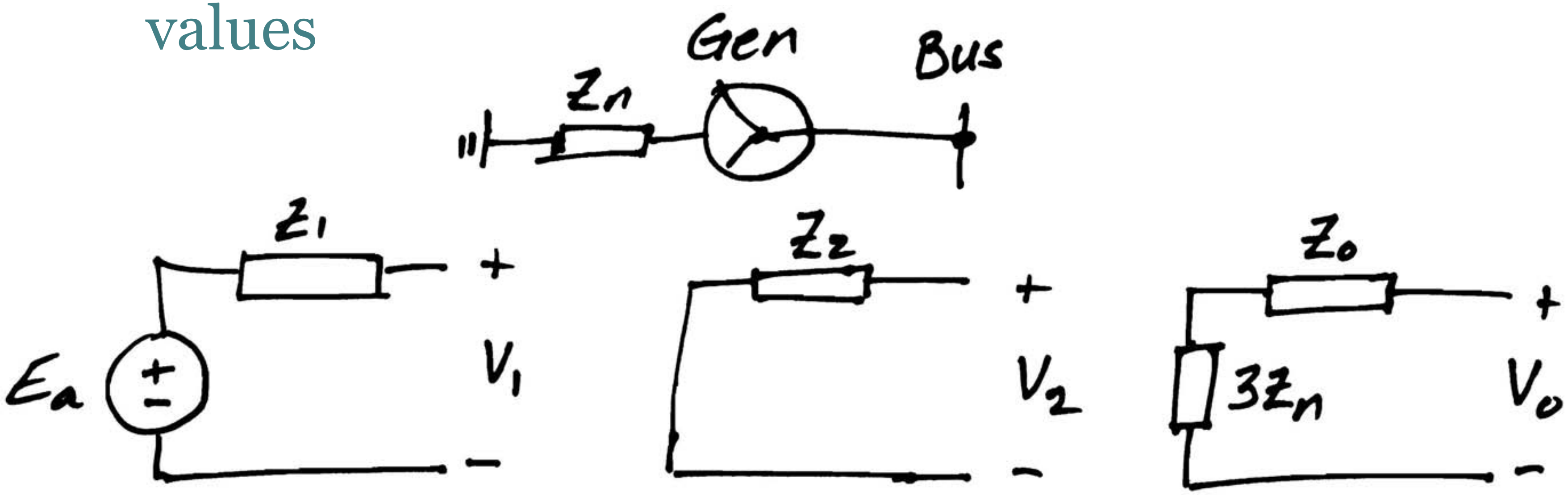
Figure 10.4 Component of the symmetrical ac current of a synchronous machine at no load where $V_{rated} = V'' = V' = V$. i''_d, i'_d, and i_d are rms values.

Generators

- Negative Sequence
  - Subtransient reactance can be measured by blocking the rotor with the field winding shorted and applying single phase voltage across any two terminals
  - As position of rotor is changed, measured reactance varies considerably if machine has salient poles without dampers (and very little damper winding exists) or if the machine has a round rotor
  - For negative sequence, similar phenomenon exists except rotor is at 2*f* with relation to field set up by applied voltage
  - Good approximation: $X_2 = \frac{1}{2}\left(X_d'' + X_q''\right)$

Generators

- Zero Sequence
  - Varies quite a lot
  - Depends largely on pitch and breadth factors of armature winding
  - Generally, X_0 is much smaller than X_1 and X_2 values



Generators

Turbine generators	Two-pole · Conventional cooled	Two-pole · Conductor cooled	Four-pole Conventional cooled	Four-pole Conductor cooled
X_d (unsat)	1.65/1.0–1.75	1.85/1.5–2.25	1.65/1.0–1.75	1.85/1.5–2.25
X_q rated current	1.61/0.96–1.71	1.81/1.46–2.21	1.61/.96–1.71	1.81/1.46–2.21
X'_d rated voltage	0.17/0.12–0.25	0.28/0.20–0.35	0.25/0.2–0.3	0.35/0.25–0.45
X''_d rated voltage	0.12/0.08–0.18	0.22/0.15–0.28	0.16/0.12–0.20	0.28/0.20–0.32
X_2 rated current	$= X''_d$	$= X''_d$	$= X''_d$	$= X''_d$
X_o rated current(1)				
x_p Potier reactance	0.07–0.17	0.2–0.45	0.12–0.24	0.25–0.45
r_2(2)	0.025–0.04	0.025–0.04	0.03–0.045	0.03–0.045
r_1(3)	0.004–0.011	0.001–0.008	0.003–0.008	0.001–0.008
r_a(3)	0.001–0.007	0.001–0.005	0.001–0.005	0.001–0.005
T'_{do}	5	5	8	6
T'_d	0.6	0.75	1.0	1.2
T''_d	0.035	0.035	0.035	0.035
T_a	0.13–0.45	0.2–0.55	0.2–0.4	0.25–0.55
H	2.5–3.5	2.5–3.5	3–4	3–4

Salient Pole Generators and Motors:

With dampers—$X'_d = 0.37/0.25–0.5$, $X''_d = 0.24/0.13–0.32$, $X_2 = X''_2$ X''_d

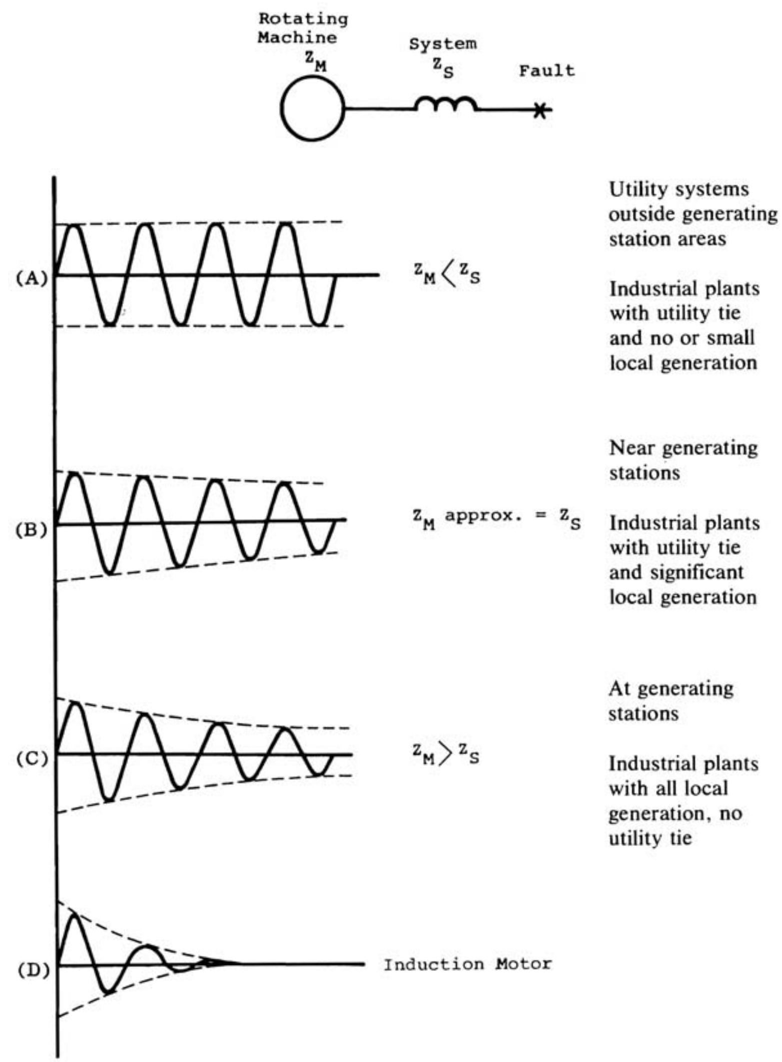
Without dampers—$X'_d = 0.35/0.25–0.5$, $X''_d = 0.32/0.20–0.5$, $X_2 = 0.4/0.3–0.45$.

Synchronous condensers:

$X'_d = 0.40$, $X''_d = 0.25$, $X_2 = 0.24$.

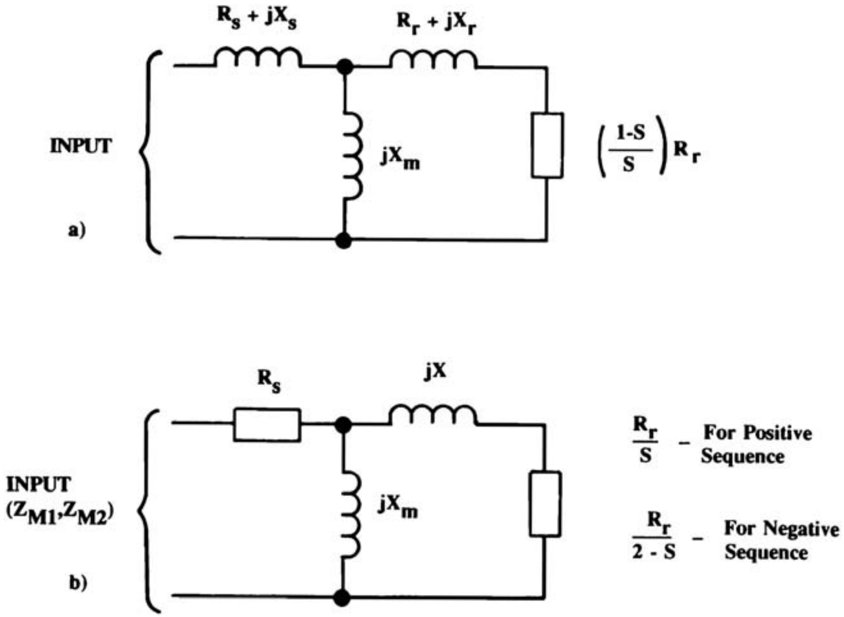
Notes: (1) X_o varies so critically with armature winding pitch that an average value can hardly be given. Variation is from 0.1 to 0.7 of X''_d; (2) r_2 varies with damper resistance; (3) r_1 and r_a vary with machine rating.

Generators & Network Equivalents



- "Strong" Remote Systems
  - Assume $X_1 = X_2$
  - Calculate X_1 from 3-phase fault duty
  - Calculate X_0 from SLG fault duty

Induction Machines



R_s = 0.01 pu = Stator Resistance

jX_s = Stator Leakage Reactance at Rated Frequency

R_r = 0.01 pu = Rotor Resistance

jX_r = Rotor Leakage Reactance at Rated Frequency

JX_m = j3.0 pu = Shunt Exciting Reactance

$jX = jX_s + jX_r = jX_d'' = j0.15$ pu

Values shown are typical for an induction motor and are per unit on the motor kVA and kV base

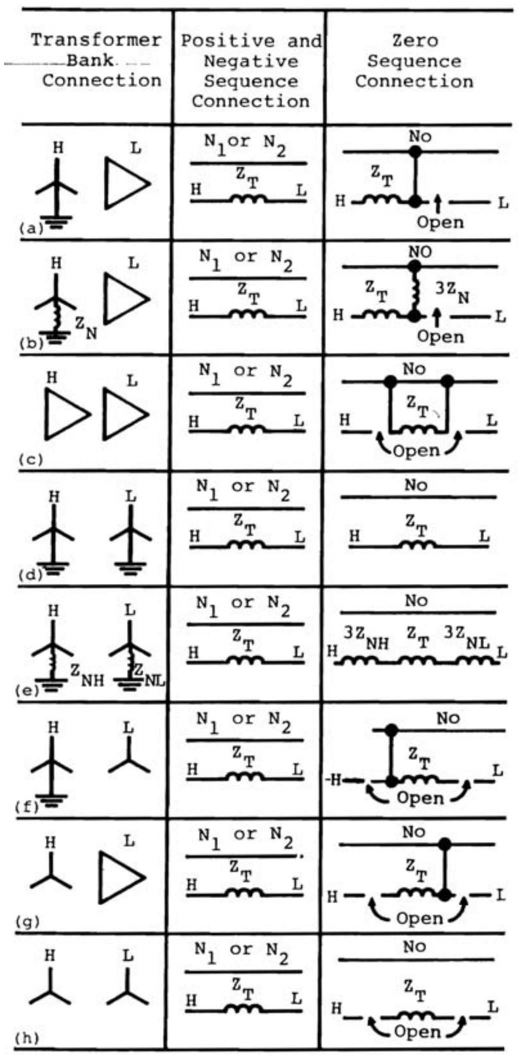
$S = \frac{\text{Synchronous RPM - Rotor RPM}}{\text{Synchronous RPM}} =$ 1.0 for stalled condition, 0+ for running conditions

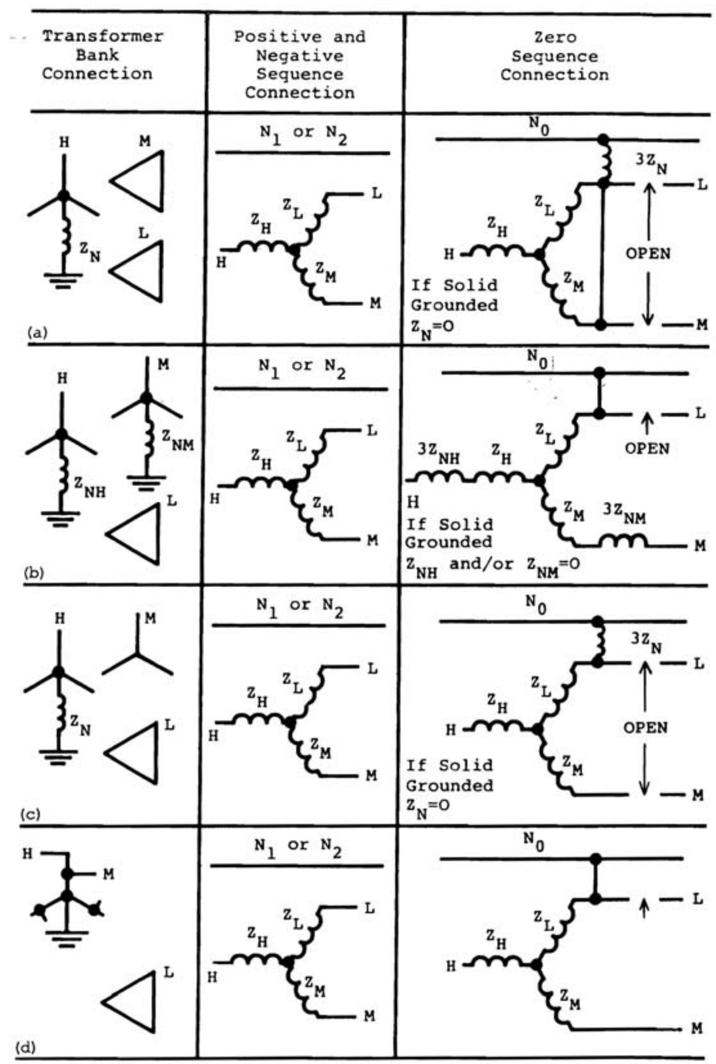
- Positive Sequence
  - Changes from stalled to running
  - ~0.15 pu stalled (X_d”)
  - 0.9–1.0 pu running
- Negative Sequence
  - Remains effectively constant
  - ~0.15 pu (X_d”)
- Zero Sequence
  - 0.0 if wye ungrounded or delta connected

Transformers

- Modelling
  - Usually modelled as a series impedance
  - Shunt parameters can be calculated by review of transformer tests.
  - Shunt parameters don't generally impact analysis
  - Transformer winding configuration determines sequence networks
  - Three winding transformers have interesting sequence networks, but close inspections shows them to be intuitive

Transformer Sequence Networks





3 Winding Transformer Impedance

- 3 Winding Transformer Impedance
  - Usually given as a winding-to-winding (delta) impedances in per cent
  - Convert to equivalent wye impedances for sequence network analysis
  - Often times, the base power is different for various impedances
  - Ex: 100 MVA auto with 35 MVA tertiary
    May show Z_{HM} on 100 MVA base
    May show Z_{HL} and Z_{ML} on 35 MVA base

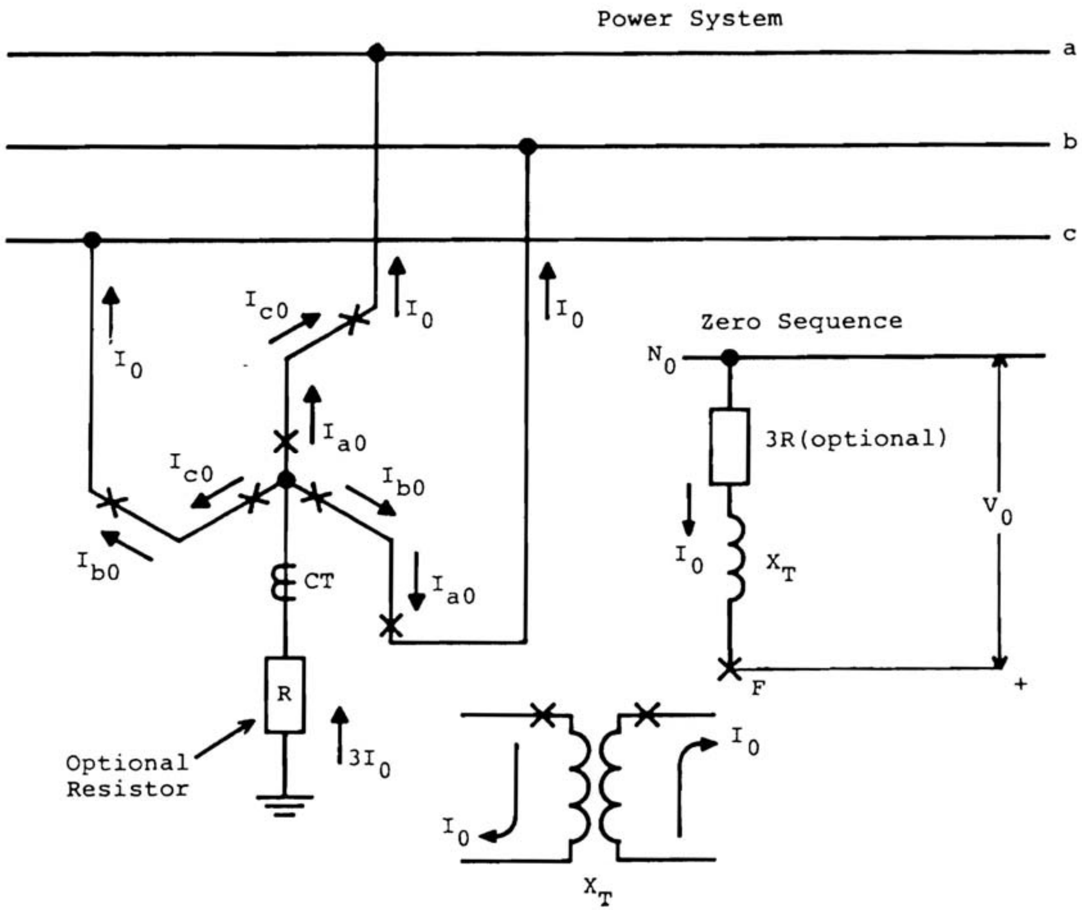
Must convert delta impedance to common base before converting to equivalent wye network

3 Winding Transformer Impedance

- Delta-Wye Conversion Formula

$$Z_H = \tfrac{1}{2}\left(Z_{HM} + Z_{HL} - Z_{ML}\right)$$
$$Z_M = \tfrac{1}{2}\left(Z_{HM} + Z_{ML} - Z_{HL}\right)$$
$$Z_L = \tfrac{1}{2}\left(Z_{HL} + Z_{ML} - Z_{HM}\right)$$

Zig-Zag Transformer



Transmission Lines

- Positive and Negative Sequence Impedance
  - Passive component
  - Assume line transpositions

$$L_1 = L_2 = \frac{\mu_0}{2\pi} \ln \frac{D_m}{D_s} \ [\mathrm{H/m}]$$

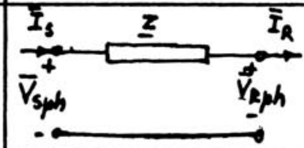
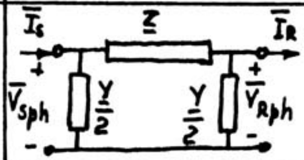
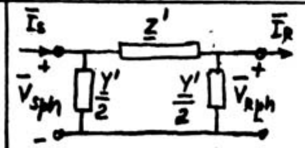
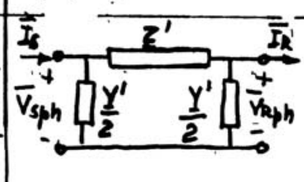
$$X = \omega L = 2\pi f L$$

$$X_1 = X_2 = 4\pi \times 10^{-7} \ln \frac{D_m}{D_s} \ [\Omega/\mathrm{m}]$$

$$R_1 = R_2 = \text{conductor AC resistance (tables)}$$

Transmission Lines

Equivalent circuit and equations for overhead lines

Line type \ characteristics	Length range [Km/mile]	$\underline{K_Z}$	$\underline{K_Y}$	Equivalent circuit (π)	Equations
short-lines	$\ell \leq$ 80/50	1	1		$\begin{cases} \bar{V}_{sph} = \bar{V}_{Rph} + \underline{Z}\,\bar{I}_R \\ \bar{I}_S = \bar{I}_R \end{cases}$
medium-length lines	80/50 $\leq \ell \leq$ 240/150	1	1		$\begin{cases} \bar{V}_{sph} = \left(1+\frac{\underline{Z}\underline{Y}}{2}\right)\bar{V}_{Rph} + \underline{Z}\,\bar{I}_R \\ \bar{I}_S = \underline{Y}\left(1+\frac{\underline{Z}\underline{Y}}{4}\right)\bar{V}_{Rph} + \left(1+\frac{\underline{Z}\underline{Y}}{2}\right)\bar{I}_R \end{cases}$
long lines	240/150 $\leq \ell$	$\frac{\sinh\sqrt{\underline{Z}\underline{Y}}}{\sqrt{\underline{Z}\underline{Y}}}$	$\frac{2(\cosh\sqrt{\underline{Z}\underline{Y}}-1)}{\sqrt{\underline{Z}\underline{Y}}\,\sinh\sqrt{\underline{Z}\underline{Y}}}$		$\begin{cases} \bar{V}_{sph} = \left(1+\frac{\underline{Z}'\underline{Y}'}{2}\right)\bar{V}_{Rph} + \underline{Z}'\,\bar{I}_R \\ \bar{I}_S = \underline{Y}'\left(1+\frac{\underline{Z}'\underline{Y}'}{4}\right)\bar{V}_{Rph} + \\ \qquad + \left(1+\frac{\underline{Z}'\underline{Y}'}{2}\right)\bar{I}_R \end{cases}$
	$\left(\begin{matrix}240/150 \\ \leq \ell \leq \\ 320/200\end{matrix}\right)^{*)}$	$1+\frac{\underline{Z}\underline{Y}}{6}+\frac{(\underline{Z}\underline{Y})^2}{120}+\cdots$	$\frac{1+\frac{\underline{Z}\underline{Y}}{12}+\frac{(\underline{Z}\underline{Y})^2}{360}+\cdots}{1+\frac{\underline{Z}\underline{Y}}{6}+\frac{(\underline{Z}\underline{Y})^2}{120}+\cdots}$		$\begin{cases} \bar{V}_{sph} = \left(1+\frac{\underline{Z}'\underline{Y}'}{2}\right)\bar{V}_{Rph} + \underline{Z}'\,\bar{I}_R \\ \bar{I}_S = \underline{Y}'\left(1+\frac{\underline{Z}'\underline{Y}'}{4}\right)\bar{V}_{Rph} + \left(1+\frac{\underline{Z}'\underline{Y}'}{2}\right)\bar{I}_R \end{cases}$

Comment: $\underline{Z}' = \underline{K_Z}\cdot\underline{Z}$; $\underline{Y}' = \underline{K_Y}\cdot\underline{Y}$; *) Particular length range for long lines.

Transmission Lines

- Zero Sequence Impedance
  - More involved
  - Assumptions
    - Zero sequence current divides equally between conductors
    - Conductors are parallel to ground
    - Earth is a solid with a plane surface, infinite in extent, and of uniform conductivity
  - None of the assumptions are true
  - We get acceptable error with these assumptions
  - Line design affects calculation techniques

Transmission Line Zero Sequence

Table 11.3 Summary for Zero-Sequence Impedance Calculations

Values are in ohms/mile at 60 Hz. For kilometers, multiply mile values by 0.62137. For other frequencies, multiply the r_e, X_a, X_d, X_e, X_g values by $f/60$, where f is the desired frequency in hertz.

Sections/ Type	General equations	Eq. no.	Equations for use with tables	Eq. no.
11.9.1[1] Single circuit, no ground wire	$Z_0 = r_a + 0.286 + j0.8382 \log \frac{D_e}{\mathrm{GMR}_{\mathrm{gr}\ 3}}$	(11.37) (11.50)	$Z_0 = r_a + r_e + j(X_a + X_e - 2X_d)$	(11.38)
11.9.3, 11.9.4[2] Single circuit, one ground wire	$Z_{0a}^{1} = r_a + j0.8382 \log \frac{\mathrm{GMD}_{\mathrm{gr}\ 3\ \mathrm{to}\ \mathrm{gr}\ 1}}{\mathrm{GMR}_{\mathrm{gr}\ \mathrm{of}\ 3}}$	(11.45)	$Z_{0a}^{1} = r_a + j(X_a + 3X_{\mathrm{d}(ag)} - 2X_d)$	(11.47)
	$Z_{0g}^{1} = 3r_g + j0.8382 \log \frac{\mathrm{GMD}_{\mathrm{gr}\ 3\ \mathrm{to}\ \mathrm{gr}\ 1}}{\mathrm{GMR}_{\mathrm{gr}\ \mathrm{of}\ 1}}$	(11.46)	$Z_{0g}^{1} = 3r_g + j(3X_g + 3X_{\mathrm{d}(ag)})$	(11.48)
	$Z_{0m} = 0.286 + j0.8382 \log \frac{D_e}{\mathrm{GMD}_{\mathrm{gr}\ 3\ \mathrm{to}\ \mathrm{gr}\ 1}}$	(11.44)	$Z_{0m} = r_e + j(X_e - 3X_{\mathrm{d}(ag)})$	(11.49)
	Total Z_0 See notes	(11.43)	Total Z_0 See notes	(11.43)
11.9.5, 11.9.6[3] Single circuit, two ground wires	$Z_{0a}^{1} = r_a + j0.8382 \log \frac{\mathrm{GMD}_{\mathrm{gr}\ 3\ \mathrm{to}\ \mathrm{gr}\ 2}}{\mathrm{GMR}_{\mathrm{gr}\ \mathrm{of}\ 3}}$	(11.53)	$Z_{0a}^{1} = r_a + j(X_a + 3X_{d(ag^1)} - 2X_d)$	
	$Z_{0g}^{1} = \frac{3r_g}{2} + j0.8382 \log \frac{\mathrm{GMD}_{\mathrm{gr}\ 3\ \mathrm{to}\ \mathrm{gr}\ 2}}{\mathrm{GMR}_{\mathrm{gr}\ \mathrm{of}\ 2}}$	(11.54)	$Z_{0g}^{1} = \frac{3r_g}{2} + j\left(\frac{3}{2} X_g + 3X_{d(ag^1)} - \frac{3}{2} X_{d(g^1)}\right)$	
	$Z_{0m} = 0.286 + j0.8382 \log \frac{D_e}{\mathrm{GMD}_{\mathrm{gr}\ 3\ \mathrm{to}\ \mathrm{gr}\ 2}}$	(11.55)	$Z_{0m} = r_e + j(X_e - 3X_{d(ag^1)})$	
	Total Z_0 See notes	(11.43)	Total Z_0 See notes	(11.43)

Transmission Line Zero Sequence

Note 1

$$\mathrm{GMR}_{\mathrm{gr\ of\ 3}} = \sqrt[9]{\mathrm{GMR}^3_{\mathrm{cond}}\, D^2_{ab} D^2_{bc} D^2_{ca}} = \sqrt[3]{\mathrm{GMR}_{\mathrm{cond}} \mathrm{GMR}^2_{\mathrm{gr\ of\ 3}}} \quad (11.50)$$

$$\text{where } \mathrm{GMD}_{\mathrm{gr\ of\ 3}}\,(X_d) = \sqrt[3]{D_{ab} D_{ba} D_{ca}} \quad (11.16)$$

Note 2

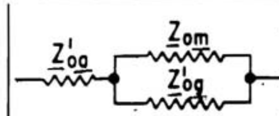
$$\mathrm{GMR}_{\mathrm{gr\ 1}} = \mathrm{GMR}_{\mathrm{gnd\ wire}}$$

$$\uparrow$$

$$\mathrm{GMD}_{\mathrm{gr\ 3\ to\ gr\ 1}}\,((X_{d(ag)}) = \sqrt[3]{D_{ag} D_{bg} D_{cg}}$$

$$\downarrow$$

$$\mathrm{GMR}_{\mathrm{gr\ 3}} = \text{same as Section 11.9.1} \quad (11.50)$$



Total Z of circuit

$$= Z^1_{0a} + \frac{Z^1_{0g} Z_{0m}}{Z^1_{0g} + Z_{0m}} \quad (11.43)$$

where

Z^1_{0a} is leakage Z of conductors $= Z_{0a} - Z_{0m}$

Z^1_{0g} is leakage Z of ground wire(s) $= Z_{0g} - Z_{0m}$

Z_{0m} is mutual Z between conductors and ground wire(s)

Z_{0a} is self Z of conductors

Z_{0g} is self Z of ground wire(s)

Note 3

$$\mathrm{GMD}_{\mathrm{gr\ 2}}\,(X_{d(g^1)})$$

$$\mathrm{GMR}_{\mathrm{gr\ 2}} = \sqrt{\mathrm{GMR}_{\mathrm{gnd\ wire}}\, D_{g^1 g^2}} \quad (11.52)$$

$$\uparrow$$

$$\mathrm{GMD}_{\mathrm{gr\ 3\ to\ gr\ 2}}\,(X_{d(ag^1)}) = \sqrt[6]{D_{ag^1} D_{bg^1} D_{cg^1} D_{ag^2} D_{bg^2} D_{cg^2}} \quad (11.51)$$

$$\downarrow$$

$$\mathrm{GMR}_{\mathrm{gr\ 3}} = \text{same as Section 11.9.1} \quad (11.50)$$

Total Z_0 of circuit

$$= Z^1_{0a} + \frac{Z^1_{0g} Z_{0m}}{Z^1_{0g} + Z_{0m}} \quad (11.43)$$

Transmission Line Zero Sequence

Table 11.3 (*Continued*)

Sections/ Type	General equations	Eq. no.	Equations for use with tables
11.9.7, 11.9.8[4] Double circuit, no ground wire	$Z_0 = r_a + 0.286 + j0.8382 \log \frac{D_e}{\mathrm{GMR}_{\mathrm{gr}\ 3}}$	(11.37) (11.50)	$Z_0 = r_a + r_e + j(X_a + X_e - 2X_d)$
	$Z_{0m} = 0.286 + j0.8382 \log \frac{D_e}{\mathrm{GMD}_{\mathrm{gr}\ 3\ \mathrm{to\ gr}\ 3}}$	(11.56)	$Z_{0m} = r_e + j(X_e - 3X_{d(s)})$
	$Z_{0(\mathrm{parallel})} = \frac{1}{2}(Z_0 + Z_{0m}) = \frac{r_a}{2} + 0.286 + j0.8382 \log \frac{D_e}{\mathrm{GMR}_{\mathrm{gr\ of}\ 6}}$	(11.58)	$Z_{0(\mathrm{parallel})} = \frac{1}{2}(Z_0 + Z_{0m}) = \frac{r_a}{2} + r_e + j\left(\frac{x_a}{2} + X_e - X_d - \frac{3}{2} X_{d(s)}\right)$
11.9.9[5] Double circuit, one ground wire	$Z_{0a}^1 = \frac{r_a}{2} + j0.8382 \log \frac{\mathrm{GMD}_{\mathrm{gr}\ 6\ \mathrm{to\ gr}\ 1}}{\mathrm{GMR}_{\mathrm{gr\ of}\ 6}}$	(11.59)	$Z_{0a}^1 = \frac{r_a}{2} + j\left(\frac{X_a}{2} + 3X_{d(a^1g)} - X_d - \frac{3}{2} X_{d(s)}\right)$
	$Z_{0g}^1 = 3r_g + j0.8382 \log \frac{\mathrm{GMD}_{\mathrm{gr}\ 6\ \mathrm{to\ gr}\ 1}}{\mathrm{GMR}_{\mathrm{gr\ of}\ 1}}$	(11.60)	$Z_{0g}^1 = 3r_g + j(3X_g + 3X_{d(a^1g)})$
	$Z_{0m} = 0.286 + j0.8382 \log \frac{D_e}{\mathrm{GMD}_{\mathrm{gr}\ 6\ \mathrm{to\ gr}\ 1}}$	(11.61)	$Z_{0m} = r_e + j(X_e - 3X_{d(a^1g)})$
	Total Z_0 See notes	(11.43)	Total Z_0 See notes or (11.43)
11.9.10, 11.9.11[6] Double circuit, two ground wires	$Z_{0a}^1 = \frac{r_a}{2} + j0.8382 \log \frac{\mathrm{GMD}_{\mathrm{gr}\ 6\ \mathrm{to\ gr}\ 2}}{\mathrm{GMR}_{\mathrm{gr\ of}\ 6}}$	(11.62)	$Z_{0a}^1 = \frac{r_a}{2} + j\left(\frac{X_a}{2} + 3X_{d(a^1g^1)} - X_d - \frac{3}{2} X_{d(s)}\right)$
	$Z_{0g}^1 = \frac{3r_g}{2} + j0.8382 \log \frac{\mathrm{GMD}_{\mathrm{gr}\ 6\ \mathrm{to\ gr}\ 2}}{\mathrm{GMR}_{\mathrm{gr\ of}\ 2}}$	(11.63)	$Z_{0g}^1 = \frac{3r_g}{2} + j\left(\frac{3}{2} X_g + 3X_{d(a^1g^1)} - \frac{3}{2} X_{d(g^1)}\right)$
	$Z_{0m} = 0.286 + j0.8382 \log \frac{D_e}{\mathrm{GMD}_{\mathrm{gr}\ 6\ \mathrm{to\ gr}\ 2}}$	(11.64)	$Z_{0m} = r_e + j(X_e - 3X_{d(a^1g^1)})$
	Total Z_0 See notes	(11.43)	Total Z_0 See notes or (11.43)

Transmission Line Zero Sequence

Note 4

GMR$_{\text{gr of 3}}$ (11.50)

$$\left\{\begin{matrix} a1 \bullet & \bullet\ c2 \\ b1 \bullet & \bullet\ b2 \\ c1 \bullet & \bullet\ a2 \end{matrix}\right\}$$

GMR$_{\text{gr of 3}}$ (11.50)

equivalent

$$\text{GMR}_{\text{gr 6}} = \sqrt{\text{GMR}_{\text{gr 3}}\,\text{GMD}_{\text{gr 3 to gr 3}}} \qquad (11.57)$$

$$\text{GMD}_{\text{gr 3 to gr 3}}\,(X_{d(s)}) = \sqrt[9]{D_{a1a2}D_{a1b2}D_{a1c2}D_{b1a2}D_{b1b2}D_{b1c2}D_{c1a2}D_{c1b2}D_{c1c2}} \qquad (11.56)$$

Note 5

$\bullet\ g\}$ GMR$_{\text{gr of 1}}$ = GMR$_{\text{gnd wire}}$ $\{\bullet g$

$$\text{GMD}_{\text{gr 6 to gr 1}}\,(X_{d(a1g)}) = \sqrt[6]{D_{a1g}D_{b1g}D_{c1g}D_{a2g}D_{b2g}D_{c2g}}$$

GMR$_{\text{gr of 3}}$ (11.50)

$$\left\{\begin{matrix} a1 \bullet & \bullet\ c2 \\ b1 \bullet & \bullet\ b2 \\ c1 \bullet & \bullet\ a2 \end{matrix}\right\}$$

GMR$_{\text{gr of 3}}$ (11.50)

GMD$_{\text{gr 3 to gr 3}}$ (same as Sections 11.9.7 and 11.9.8)

GMR$_{\text{gr 6}}$ (11.57)

Total Z_0 of circuit

$$= Z^1_{0a} + \frac{Z^1_{0g}Z_{0m}}{Z^1_{0a} + Z_{0m}} \qquad (11.43)$$

Note 6

$\bullet\ g_1 \quad \bullet g_2\}$ GMR$_{\text{gr of 2}}$ (11.52) $\{$ GMR$_{\text{gr of 2}}$ (11.52)

$$\text{GMD}_{\text{gr 6 to gr 2}}\,(X_{d(a1g1)}) = \sqrt[12]{D_{a1g1}D_{b1g1}D_{c1g1}D_{a2g1}D_{b2g1}D_{c2g1}D_{a1g2}D_{b1g2}D_{c1g2}D_{a2g2}D_{b2g2}D_{c2g2}}$$

GMR$_{\text{gr of 3}}$ (11.50)

$$\left\{\begin{matrix} a1 \bullet & \bullet\ c2 \\ b1 \bullet & \bullet\ b2 \\ c1 \bullet & \bullet\ a2 \end{matrix}\right\}$$

GMR$_{\text{gr of 3}}$ (11.50)

GMD$_{\text{gr 3 to gr 3}}$ = same as Sections 11.9.7 and 11.9.8

GMR$_{\text{gr 6}}$ (11.57)

Total Z_0 of circuit

$$= Z^1_{0a} + \frac{Z^1_{0g}Z_{0m}}{Z^1_{0g} + Z_{0m}} \qquad (11.43)$$

Transmission Line Zero Sequence

The various values are summarized as follows:

r or r_a = ac resistance of the phase conductors

r_g = ac resistance of the ground wire(s)

r_e = 0.286 Ω/mile at 60 Hz

X_a or $X_g = 0.2794 \log \frac{1}{\text{GMR}}$, ohms/mile at 60 Hz

$X_d = 0.2794 \log \text{GMD}$, ohms/mile at 60 Hz

$X_e = 0.8382 \log D_e$, ohms/mile at 60 Hz

$\text{GMR} = \text{GMR}_a = \text{GMR}_{\text{cond.}}$ = conductor geometric mean radius, feet

$\text{GMR}_g = \text{GMR}_{\text{gnd wire}}$ = ground wire geometric mean radius, feet

Otherwise, the GMR values are specified for various groups

GMD = geometric mean distance between groups as specified

D_e = equivalent depth of earth return

D_{ab}, D_{ag}, etc. = distances between respective conductors or ground wires, feet

What We Won't Cover

- What We Won't Cover
  - Capacitive Reactance of Transmission Lines
  - Mutual Impedance of Transmission Lines
  - Cable Modelling (look up in table or contact manufacturer)
- So What's the General Idea?
  - Produce a positive, negative, and zero sequence model for your components.
  - Simple model for equipment is a series impedance
  - Connect the models according to system topology
  - Analyze imbalances by connecting systems

Faults & Sequence Networks

Faults

- Definition
  - A fault is any condition in a system considered abnormal
  - Basic Types
    - Short-Circuit Faults
    - Open-Circuit Faults
    - Combined Faults
- Shunt Faults
  - Three-Phase (3Ph)
  - Phase-To-Phase (LL)
  - Phase-To-Phase-To-Ground (LLG)
  - Single-Line-To-Ground (SLG)

Faults

- Series Faults
  - Open phase
  - Open neutral
  - Two open phases
  - Impedance in a phase
- Combination Faults
  - See connection diagrams

Sequence Connections – Shunt Faults

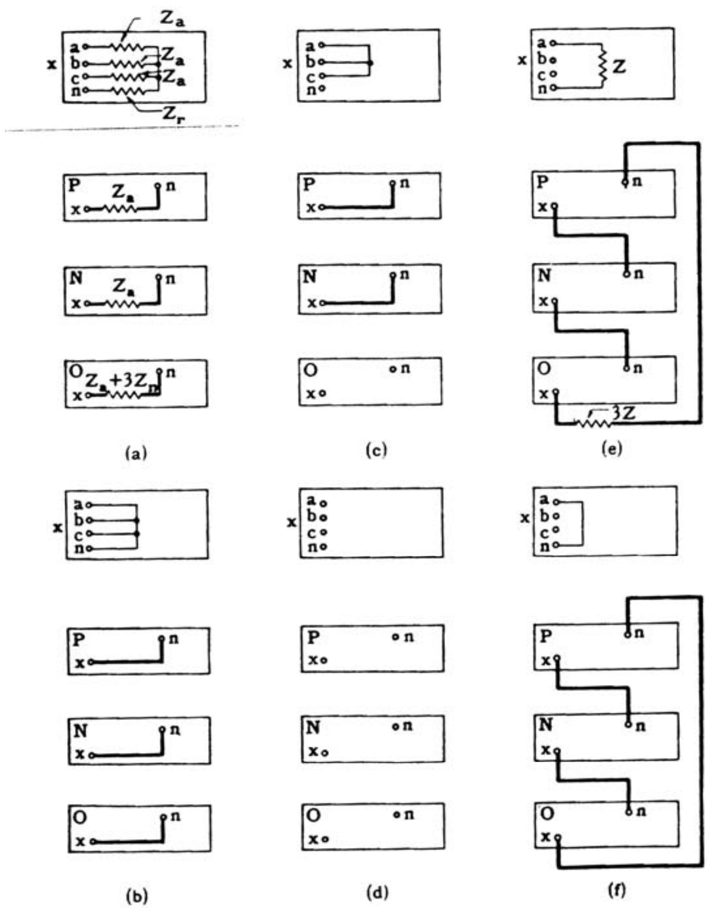
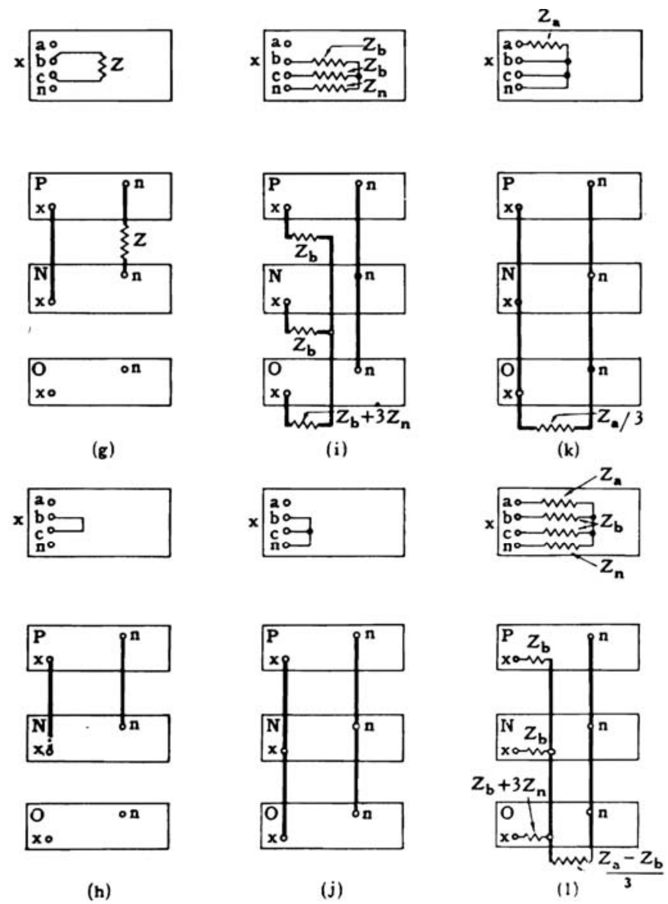


Figure 5.3 Box sequence connections for shunt balanced and unbalanced conditions: (a) balanced load or three-phase-to-ground fault with impedances; (b) three-phase-to-ground fault; (c) three-phase fault; (d) shunt circuit open; (e) phase-to-ground fault through an impedance; (f) phase-to-ground fault; (g) phase-to-phase fault through impedance; (h) phase-to-phase fault; (i) two-phase-to-ground fault through impedance;

Sequence Connections – Shunt Faults



(j) two-phase-to-ground fault; (k) three-phase-to-ground fault with impedance in phase *a*; (l) unbalanced load or three-phase-to-ground fault with impedance. (From E. L. Harder, Sequence Network Connections for Unbalanced Load and Fault Conditions, *The Electrical Journal*, December 1937.)

Sequence Connections – Series Faults

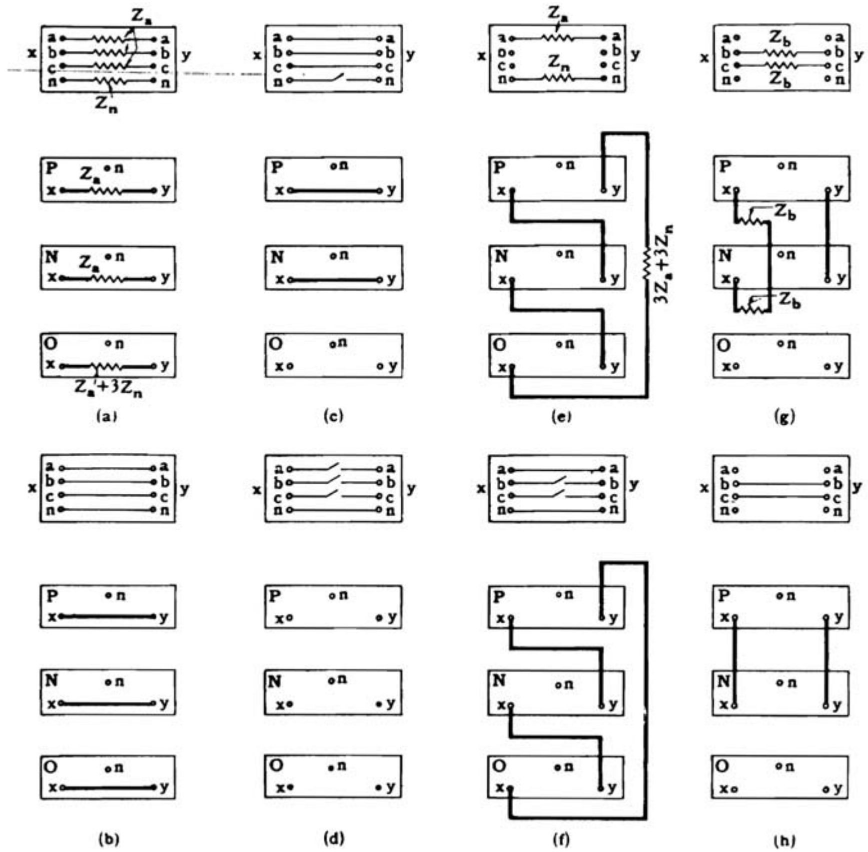
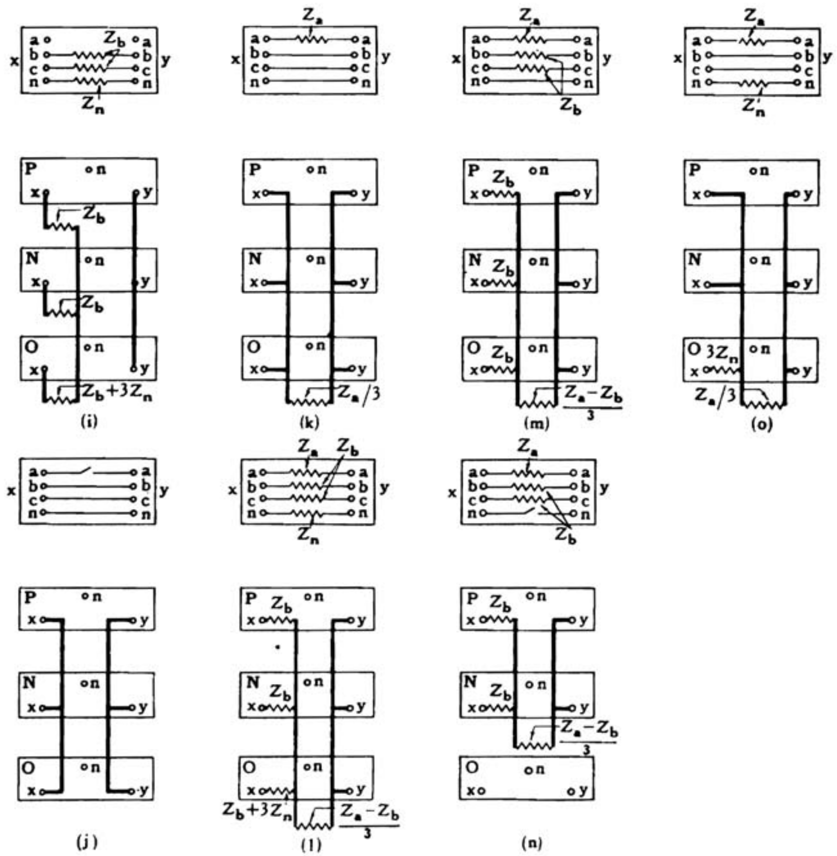


Figure 7.1 Box sequence connections for series unbalanced conditions: (a) equal impedances in three phases; (b) normal balanced conditions; (c) neutral circuit open; (d) any three or four phases open; (e) phases *b* and *c* open and impedances in phase *a* and neutral; (f) phases *b* and *c* open; (g) phases *a* and neutral open and impedances in phases *b* and *c*; (h) phase *a* and neutral open; (i) phase *a* open and impedances

Sequence Connections – Series Faults



in phases b, c, and neutral; (j) phase a open; (k) impedance in phase a; (l) equal impedances in phases b and c, impedance in neutral; (m) equal impedances in phases b and c; (n) equal impedances in phases b and c, neutral open; (o) impedances in phase a and neutral. (From E. L. Harder, Sequence Network Connections for Unbalanced Load and Fault Conditions, *The Electrical Journal*, December 1937.)

Sequence Connections – Combination Faults

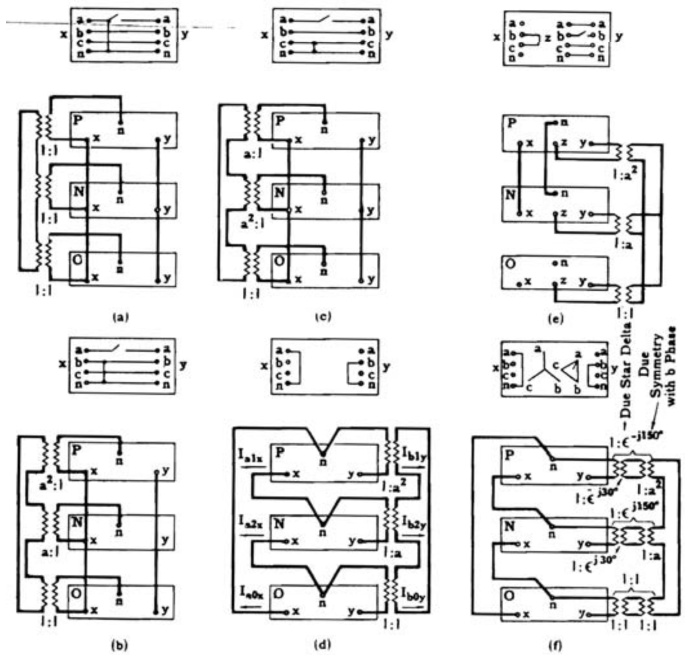
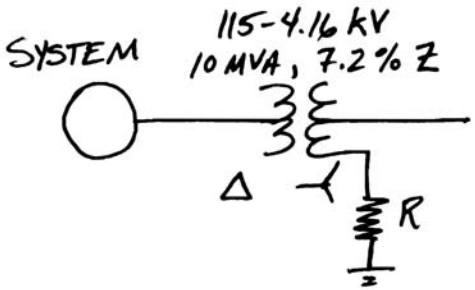
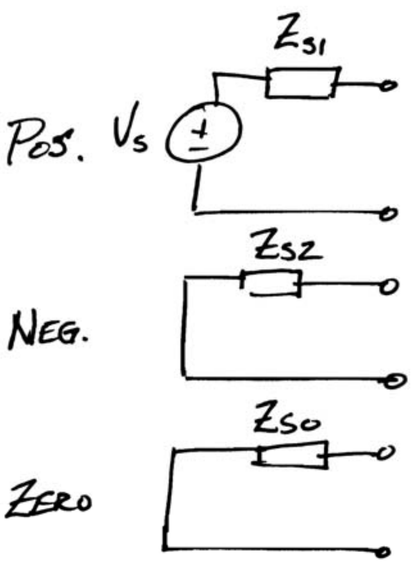


Figure 7.5 Box sequence connections for simultaneous unbalanced conditions: (a) phase *a* open and phase-*a*-to-ground fault on the *x* side; (b) phase *a* open and phase-*b*-to-ground fault; (c) phase *a* open and phase-*c*-to-ground fault; (d) phase-*a*-to-ground fault at *x* and phase-*b*-to-ground fault at *y*; (e) phase-*b*-to-*c* fault at *x* and phase *b* open *z* to *y*; (f) phase-*a*-to-neutral fault at *x*, phase-*b*-to-neutral fault on the other side of a wye–delta transformer bank at *y*, with *x* taken as the reference point. (From E. H. Harder, Sequence Network Connections for Unbalanced Load and Fault Conditions, *The Electrical Journal*, December 1937.)

Example 1 – System Parameters



SYSTEM MODEL



SYSTEM

3φ FAULT: 29 kA ; X/R = 15

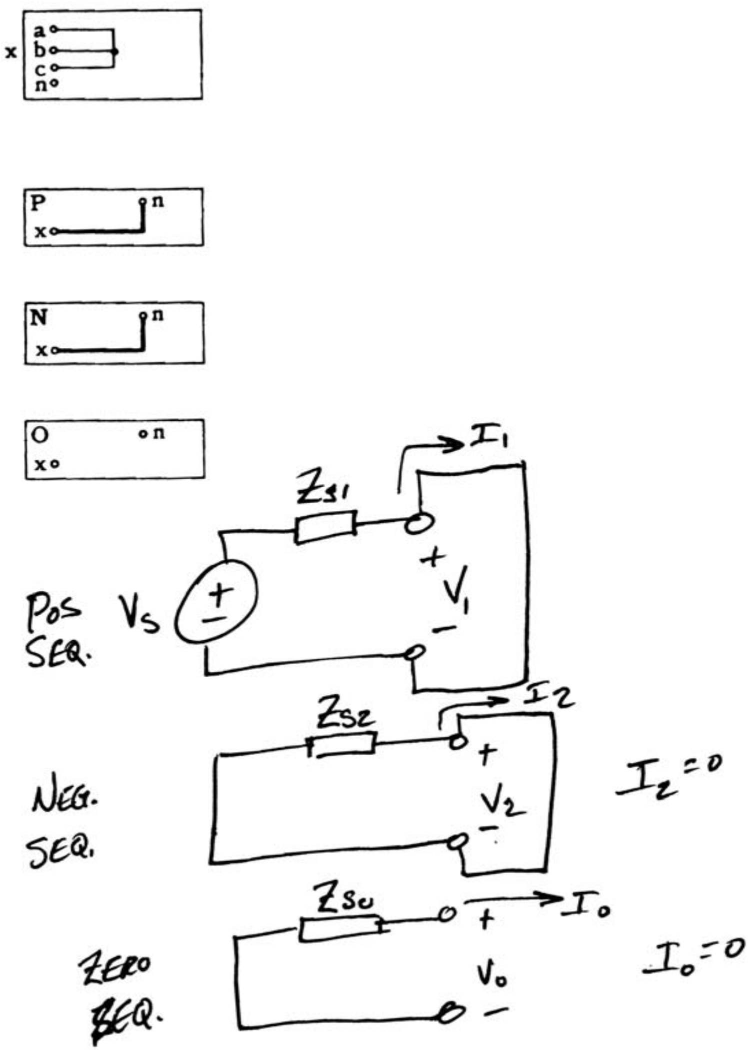
SLG FAULT: 23 kA ; X/R = 8.2

BASE QUANTITIES

1. CHOOSE $S_b = 10$ MVA
2. CHOOSE $V_b = 115$ kV
3. $$I_b = \frac{10{,}000 \text{ kVA}}{\sqrt{3}\,(115 \text{ kV})} = 50.2\,[A]$$
4. CHOOSE $V_b = 4.16$ kV

$$I_b = \frac{10{,}000 \text{ kVA}}{\sqrt{3}\,(4.16 \text{ kV})} = 1387.9\,[A]$$

Example 1 – System Parameters



3-PHASE FAULT

VOLTAGE REF = 0°

$86.2 = \tan^{-1}(15)$

$$I_a = \frac{29{,}000 \angle -86.2^\circ \text{ [A]}}{50.2 \text{ [A]}} = 577.6 \angle -86.2 \text{ pu}$$

$$I_b = 577.6 \angle 153.8 \text{ pu}$$

$$I_c = 577.6 \angle -33.8 \text{ pu}$$

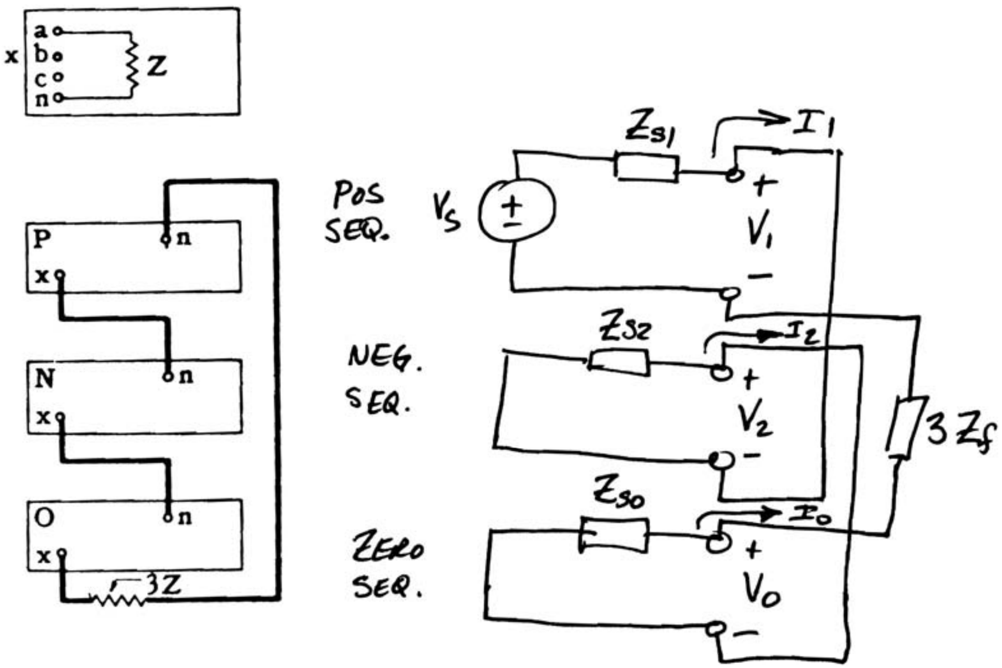
$$I_{sym} = T^{-1} \begin{bmatrix} 577.6 \angle -86.2 \\ 577.6 \angle 153.8 \\ 577.6 \angle -33.8 \end{bmatrix} = \begin{bmatrix} 0 \\ 577.6 \angle -86.2 \\ 0 \end{bmatrix}$$

$$Z_{S1} = \frac{V_S}{I_1} = \frac{1 \angle 0^\circ}{577.6 \angle -86.2}$$

$$= 0.00173 \angle 86.2 \text{ [pu]}$$

$$Z_{S2} = Z_{S1}$$

Example 1 – System Parameters



SLG FAULT

$-V_s + I_1 Z_{s1} + I_2 Z_{s2} + I_0 Z_{s0} + 3I_0 Z_f = 0$

$V_s = I_0 (Z_{s1} + Z_{s2} + Z_{s0} + 3Z_f)$

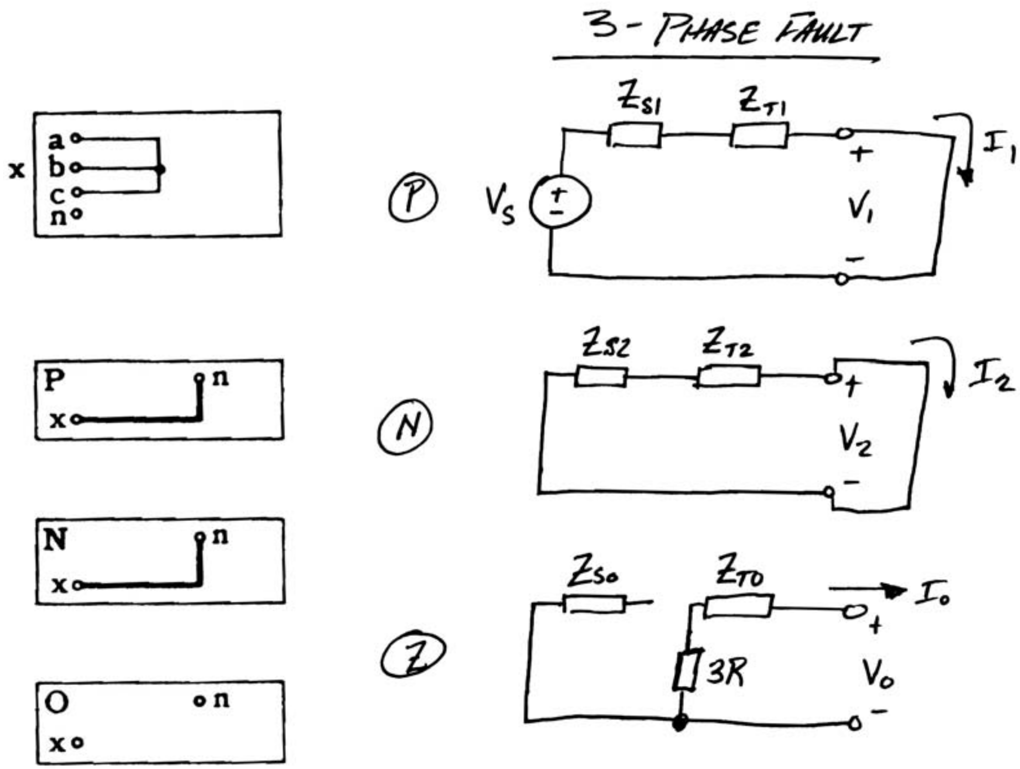
Assume $Z_f = 0$ & $Z_{s1} = Z_{s2}$

$\frac{V_s}{I_0} = Z_{s0} + 2Z_{s1}$

$Z_{s0} = \frac{V_s}{I_0} - 2Z_{s1}$

$= 0.00310 \angle 79.4^\circ$ [pu]

Example 2 – Delta Wye Transformer 3-Ph Fault



Base Quantities

$S_b = 10$ MVA

$V_b = 115$ kV & 4.16 kV

Transformer Impedance

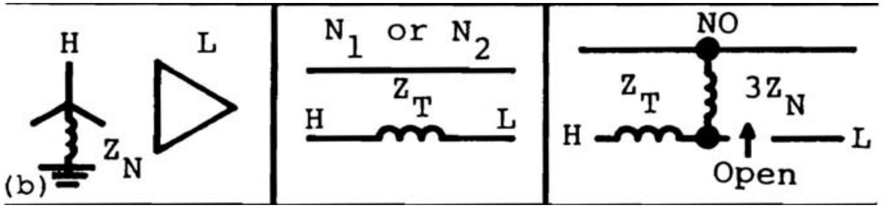
For simplicity

$Z_{T1} = Z_{T2} = Z_{T0} = j0.072$ pu

Calculations

$I_0 = I_2 = 0$

$$I_1 = \frac{V_s}{Z_{S1} + Z_{T1}} = \frac{1\angle 0^\circ \text{ pu}}{0.00173\angle 86.2 +}$$



Example 2 – Delta Wye Transformer 3-Ph Fault

Calculations

$$I_0 = I_2 = 0$$

$$I_1 = \frac{V_S}{Z_{S1} + Z_{T1}} = \frac{1\angle 0^\circ}{0.00173\angle 86.2 + 0.072\angle 90^\circ}$$

$\leftarrow$ Neglect

$$\approx \frac{1\angle 0^\circ}{0.072\angle 90^\circ}$$

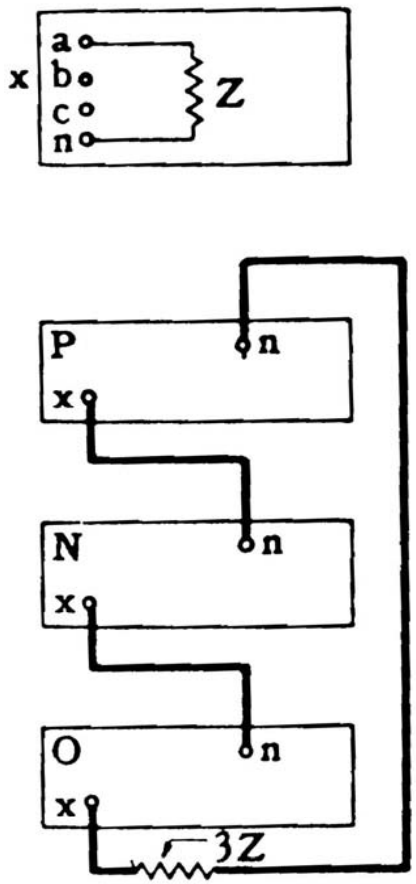
} Shows 3-phase fault as relation to transf. Z

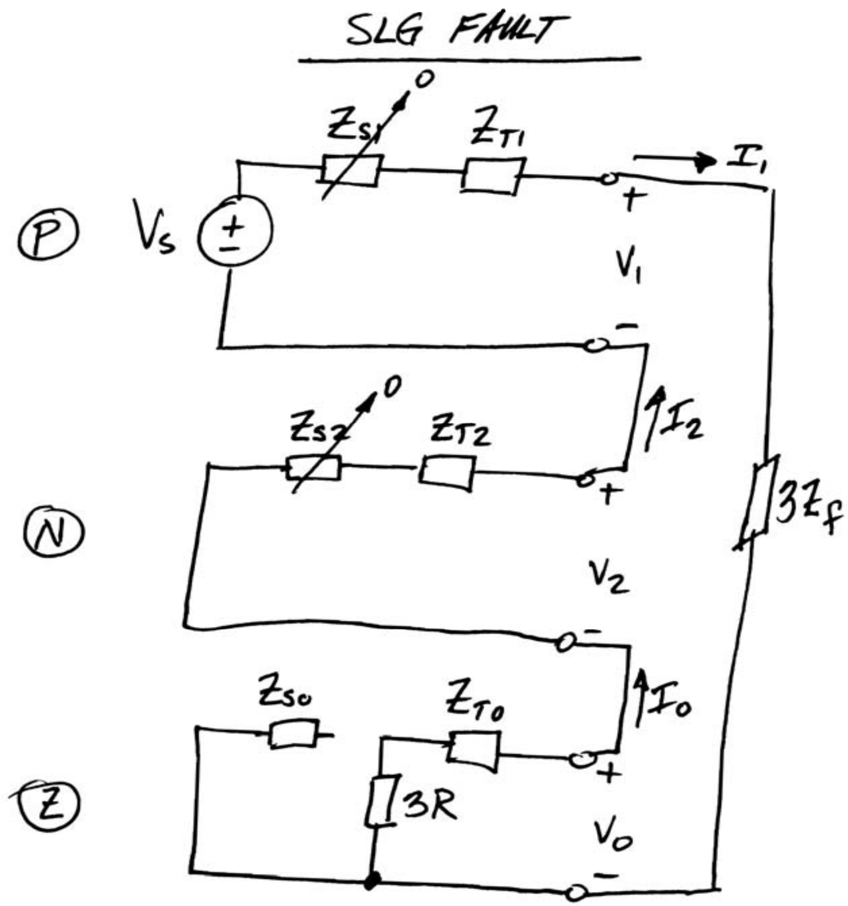
$$I_1 = 13.89\angle -90^\circ \text{ [pu]}$$

$\leftarrow$ 13.89 times rated current

$$I_{ph} = T \begin{bmatrix} 0 \\ 13.89\angle -90^\circ \\ 0 \end{bmatrix} = \begin{bmatrix} 13.89\angle -90^\circ \\ 13.89\angle 150^\circ \\ 13.89\angle 30^\circ \end{bmatrix} \text{ [pu]}$$

Example 3 – Delta Wye Transformer SLG Fault





Example 3 – Delta Wye Transformer SLG Fault

Calculations

$$-V_s + I_1 Z_{T1} + I_0(3Z_f) + I_0(3R + Z_{T0}) + I_2 Z_{T2} = 0$$

$$I_1 = I_2 = I_0$$

$$Z_{T1} = Z_{T2} = Z_{T0} = Z_T$$

$$\cancel{V_s = 3(Z_T + R + Z_f)}$$

$.T$

$$V_s = 3I_0(Z_T + R + Z_f)$$

Assume $R = Z_f = 0$

$$I_0 = \frac{V_s}{3Z_T} = \frac{1\angle 0^\circ}{(3)(0.072\angle 90^\circ)}$$

$$= 4.63\angle -90^\circ \; [pu]$$

$$I_{ph} = T\begin{bmatrix} 4.63\angle -90^\circ \\ 4.63\angle -90^\circ \\ 4.63\angle -90^\circ \end{bmatrix} = \begin{bmatrix} 13.89\angle -90^\circ \\ 0 \\ 0 \end{bmatrix} [pu]$$

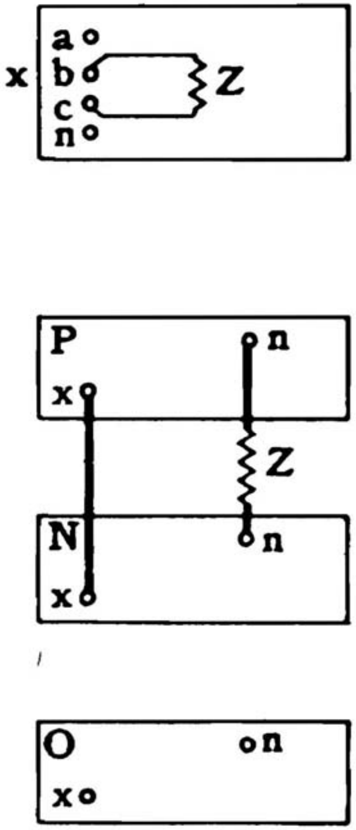
Primary Side Currents

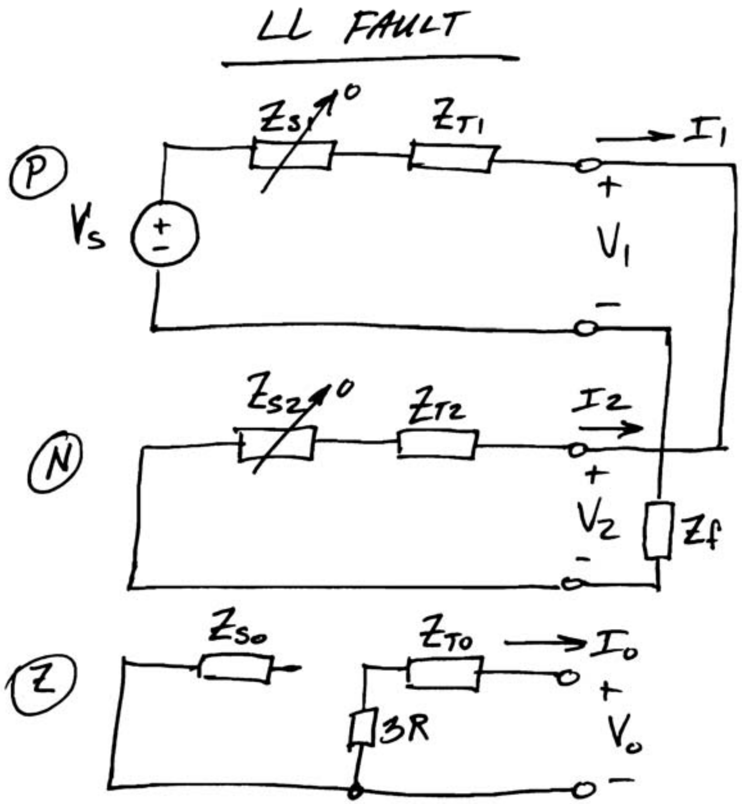
$$\left.\begin{array}{l} \text{Shift } I_1 \text{ by } +30^\circ \\ \text{Shift } I_2 \text{ by } -30^\circ \end{array}\right\} \text{Effect of } \Delta\text{-}\curlyvee \text{ transformer}$$

$$I_{ph} = T\begin{bmatrix} 0 \\ 4.63\angle -60^\circ \\ 4.63\angle -120^\circ \end{bmatrix} = \begin{bmatrix} 8.02\angle -90^\circ \\ 0 \\ 8.02\angle 90^\circ \end{bmatrix}$$

$$\frac{8.02}{13.89} = 0.577 = \frac{1}{\sqrt{3}}$$

Example 4 – Delta Wye Transformer LL Fault





Example 4 – Delta Wye Transformer LL Fault

Calculations

$$-V_S + I_1(\cancel{Z_{S1}}^{0} + Z_{T1}) - I_2(\cancel{Z_{S2}}^{0} + Z_{T2}) + I_1 \cancel{Z_f}^{0} = 0$$

$$V_S = I_1(Z_{T1} + Z_{T2}) \longleftarrow Z_T = Z_{T1} = Z_{T2}$$

$$I_1 = \frac{V_S}{2Z_T} = \frac{1\angle 0^\circ}{2(0.072\angle 90^\circ)}$$

$$I_1 = 6.94\angle -90^\circ \text{ [pu]}$$

$$I_2 = 6.94\angle 90^\circ \text{ [pu]}$$

$$\frac{6.94}{13.89} = \frac{1}{2}$$

$$I_{ph} = T\begin{bmatrix} 0 \\ 6.94\angle -90^\circ \\ 6.94\angle 90^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ 12.03\angle 180^\circ \\ 12.03\angle 0^\circ \end{bmatrix} \text{ [pu]}$$

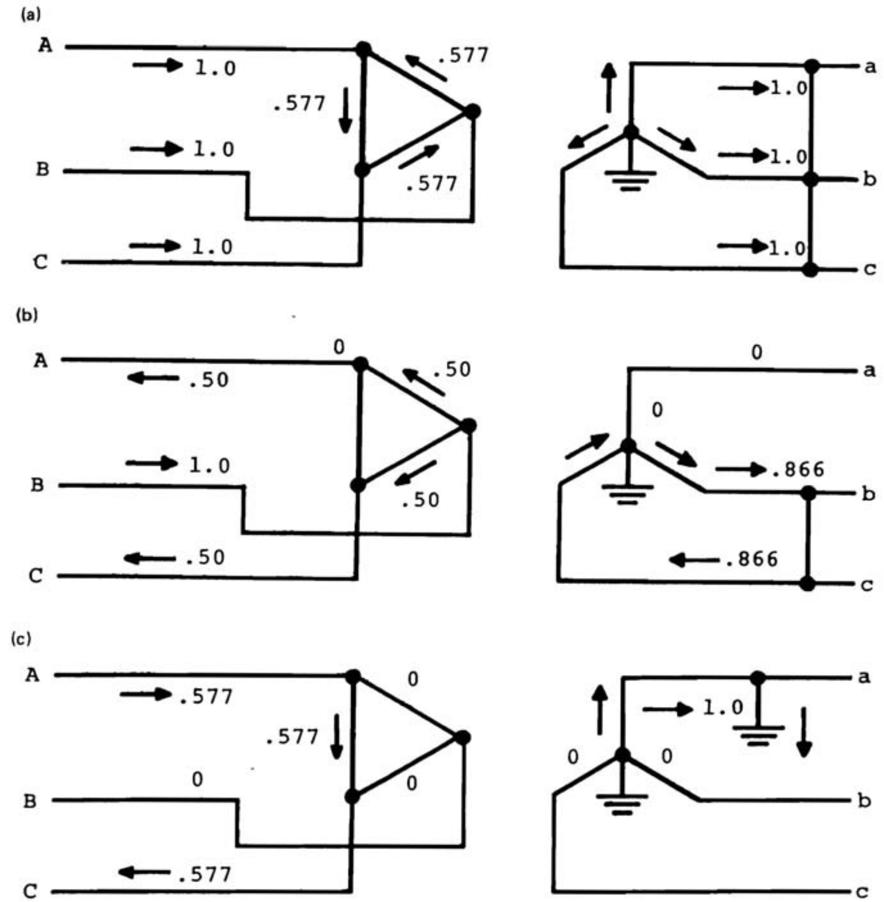
$$\frac{12.03}{13.89} = 0.866 = \frac{\sqrt{3}}{2}$$

Primary Side Currents

$$I_{ph} = T\begin{bmatrix} 0 \\ 6.94\angle -60^\circ \\ 6.94\angle 60^\circ \end{bmatrix} = \begin{bmatrix} 6.94\angle 0^\circ \\ 13.89\angle 180^\circ \\ 6.94\angle 0^\circ \end{bmatrix} \text{ [pu]}$$

Delta-Wye Summary

- 3-Phase Fault
  - Behaves as expected, same current both sides
- LL Fault
  - Must coordinate full fault current on primary with 0.866 factor on secondary (i.e. coordination interval)
- SLG Fault
  - For solidly grounded system, shift damage curve by 0.577 to protect against full fault current on secondary



Example 5 – Resistor Effect

RESISTOR ON TRANSFORMER NEUTRAL

EX: 200 A RESISTOR

2400 V

$$\Rightarrow R = \frac{2400}{200} = 12\,[\Omega]$$

$$Z_b = \frac{(4.16\text{ kV})^2}{10\text{ MVA}} = 1.73\,[\Omega]$$

$$R_{pu} = \frac{12\,[\Omega]}{1.73\,[\Omega]} = 6.93\,[pu]$$

$$-V_S + I_1 Z_{T1} + I_0(3Z_f) + I_0(3R + Z_{T0}) + I_2 Z_{T2} = 0$$

$$I_0 = I_1 = I_2$$

$$Z_T \triangleq Z_{T1} = Z_{T2} = Z_{T0}$$

$$V_S = 3I_0(Z_T + R + Z_f)$$

$$Z_f = 0$$

$$Z_T = j0.072\,[pu]$$

$R = 6.93\,[pu] \longleftarrow$ Dominant term

$$I_0 = \frac{V_S}{3(Z_T + R + Z_f)} = \frac{1\angle 0^\circ}{3(6.93 + j0.072)}$$

$$I_0 = 0.048\angle -0.6^\circ\,[pu]$$

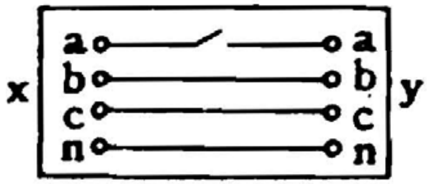
$$I_n = 3I_0 = 0.144\angle -0.6^\circ\,[pu]$$

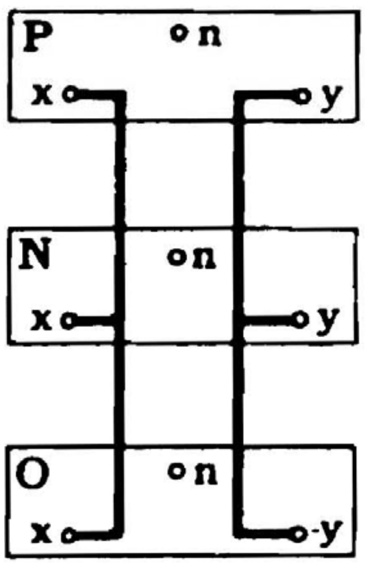
$$I_b = 1387.9\,[A]$$

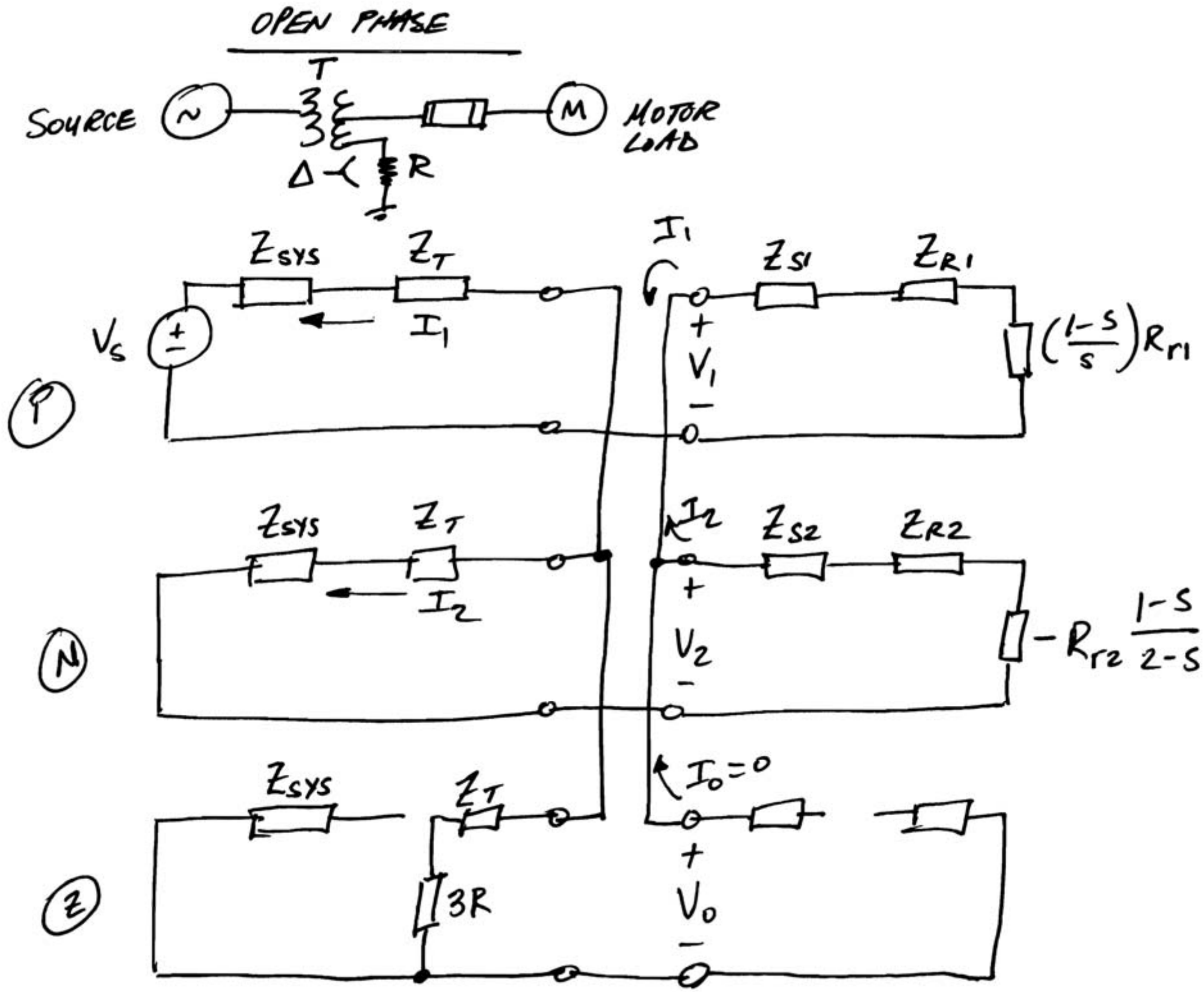
$$\Rightarrow I_n = (0.144\angle -0.6^\circ\,[pu])(1387.9\,[A])$$

$$= \underline{\underline{200\angle -0.6^\circ\,[A]}}$$

Example 6 – Open Phase Fault



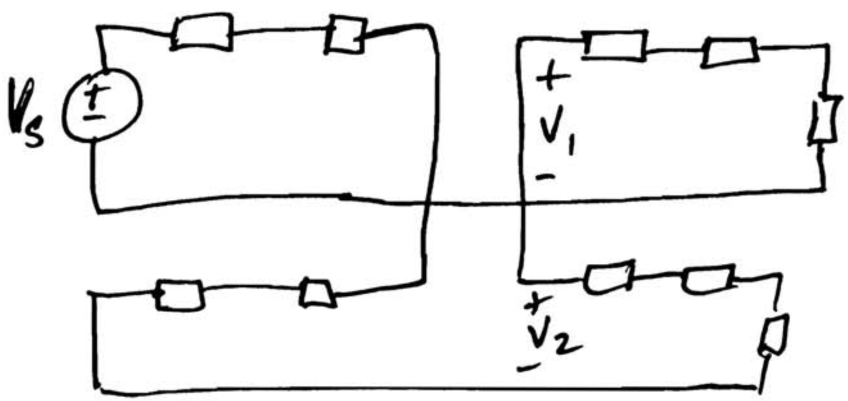




Example 6 – Open Phase Fault

$$I_1 = -I_2$$

$$-V_S - I_1(Z_{SYS} + Z_T) - I_1(Z_{SYS} + Z_T) + I_1\left(R_{r2}\frac{1-S}{2-S}\right) - I_1(Z_{S2} + Z_{R2})$$
$$- I_1(Z_{S1} + Z_{R1}) - I_1\left(\frac{1-S}{S}\right)R_{r1} = 0$$



Questions?

Thank You